Abstract: A reduced-complexity aeolian dune stratification model is developed and applied to explore the role of dune morphodynamics in the creation of synthetic sections of aeolian stratigraphy originating from three sets of environmental forcing: 1) steady wind transport capacity, 2) steady bed aggradation and variable wind transport capacity, and 3) steady wind transport capacity and bed aggradation. In each scenario, the forward motion of initial, highly disorganized dunes generates a significant record exclusively containing autogenic signals that arise from early dune growth, deformation, and merging. However, continued dune growth scours deeply, and shreds all records of early dunes. Afterward, dunes self-organize into quasi-stable groups. Forward motion of dune groups creates, truncates, and amalgamates sets and co-sets of cross-strata, quickly forming a second, significantly more robust stratigraphic record, which preserves a comingling of signals sourced from ongoing autogenic processes and each scenario’s specific set of environmental forcings, the allogenic boundary conditions of the sand sea. Although the importance of self-organization on modeled aeolian stratification is clear in the few presented scenarios, self-organization may be throttled via variability in environmental forcings, as thoroughly documented in a companion paper (Cardenas et al., this issue). Therefore, additional work is warranted because this numerical experiment only begins to sample possible sets of environmental forcing, boundary conditions, and initial conditions, geomorphic responses, and consequential preservation possible in the presented surface-stratigraphic bedform modeling framework.

Introduction

Aeolian dune fields emerge and form patterns by autogenic processes operating within a set of allogenic boundary conditions, which, broadly speaking, are ultimately derived from climatic, tectonic, and base-level basin-scale processes (Kocurek 1999; Jerolmack and Paola 2010; Rodriguez-López et al. 2014). A fundamental challenge of stratigraphy, and a common task of workers, is to unravel the interplay of autogenic and allogenic signals frozen in sections of aeolian sedimentary rock to reconstruct the morphology of ancient dunes and the allogenic conditions that existed in the ancient environment (Eastwood et al. 2012). However, in a dry sand sea composed of readily erodible sediments, the tumultuous motion of dunes may cause punctuated, non-uniform scouring and filling in the sediment accumulation, plausibly cannibalizing previously deposited material and shredding environmental signals. To explore the interplay of autogenic and allogenic processes, and incompleteness of the aeolian rock record, a reduced-complexity model of formation of bedform strata is extended from existing models of bedform topography (Jerolmack and Mohrig 2005a; Swanson et al. 2017) and applied to explore the role of dune morphodynamics in the creation of a synthetic aeolian rock record and shedding of environmental signals originating from three sets of allogenic boundary conditions. In a companion article in this issue, Cardenas et al. (2019), present detailed mapping of set-scale architecture of the Jurassic Page Sandstone, which is used to similarly parse the relative contributions of competing autogenic and allogenic processes.

The creation and preservation of an aeolian rock record relies on several environmental factors. Firstly, sufficient sediment must be made available and transported to allow sand sea construction. Secondly, sediment accumulation occurs if appropriate spatial and temporal changes in sediment transport capacity allow the formation of a sedimentary body. And finally, accommodation is needed to preserve this sediment accumulation (Kocurek 1999). Environmental signals encoded in the aeolian rock record arise from changes in external forcings, i.e., the allogenic boundary conditions of the sand sea, such as sediment supply and annual cyclicity of sediment-transporting wind (Rubin 1987; Eastwood et al. 2012; Ping et al. 2014; Courrech du Pont et al. 2014; Swanson et al. 2016), and areal extent of sandsea development (Ewing and Kocurek 2010). Although direct linkages between changes in environmental forcing and dune-pattern response are not entirely understood, an unsteady external forcing is thought to drive geomorphic responses (Ewing and Kocurek 2010), such as changes in dune size, shape, spacing, and motion, which, if preserved, are encoded as spatial variation in the geometry and arrangement of inclined strata and truncation surfaces that make up aeolian architecture.

Recent studies have identified architectural elements in aeolian sections that arise from dune autogenic processes known as bedform interactions (Brothers et al. 2017; Day and Kocurek 2017): the way individual...
bedforms collide, merge, split, or otherwise interact in the context of dune-pattern formation (Kocurek et al. 2010). Although representing only a subset of unsteady dune motion imparted by dune processes, this substantial progress toward linking autogenic dune processes to aeolian stratigraphy highlights a need for tractable hypotheses that provide workers with testable linkages between autogenic and allogenic processes, and the aeolian rock record (Rodríguez-López et al. 2014). Ideally, these hypotheses would arise from observing modern dune fields and their recent deposits (Brothers et al. 2017). However, due to the vast time and spatial scales of aeolian systems, a viable alternative is to implement a forward model of bedform stratification to explore the roles of dune morphodynamics and environmental forcing in the creation of a synthetic aeolian rock record. However, limitations exist in forward models of bedform stratigraphy. For example, while the geometric model of Rubin (1987) provides a tool to forward model aeolian architecture arising from the motion of dunes using an interpreted or assumed dune morphology, it does not include autogenic or allogenic processes. Similarly, the bedform stratification model of Jerolmack and Mohrig (2005b) uses a continuum granular-flow model to reproduce stratigraphy that arises from the forward motion of deforming bedforms, but does not resolve initial bedform growth or geomorphic response to changes in environmental forcing. Therefore, a reduced-complexity model of one-dimensional (1D) bedform topography is modified to create two-dimensional (2D) vertical sections of synthetic stratigraphy that encode information from both autogenic processes and imposed allogenic conditions. This surface-stratigraphic model is used to conduct a set of numerical experiments, varying (allogenic) boundary conditions of transport capacity and bed aggradation. The experimental results suggest that the earliest record of dune-field growth is eroded by continued dune (autogenic) self-organization into long-wavelength, low-amplitude groups. After group formation, rates of dune self-organization wane, processes operating in the dune field become sensitive to environmental forcing, and con mingled autogenic and environmental signals propagate into the synthetic rock record.

**METHODS**

**Bedform Strata-Formation Model**

The bedform strata-formation model adopts the bedform surface modeling strategy of Jerolmack and Mohrig (2005a) and Swanson et al. (2017), which casts bedform growth and motion as the deformation of a dynamic boundary between sediment and its transporting fluid, but does not resolve the fluid flow field. The motion of this boundary is driven by morphodynamic feedbacks between bedform topography, $\eta$, bed shear stress, $\tau_b$, and saturated sediment flux, $q_s$. This feedback is formed by (1) casting $q_s$ as a power-law function of $\tau_b$ (Meyer-Peter and Müller 1948), (2) expressing $\tau_b$ as a function of $\eta$, and (3) calculating temporal change in $\eta$ as a consequence of spatial change in $q_s$. This system reproduces the fundamental morphodynamic behavior of bedforms including growth, interaction, and eventual self-organization to a dynamic-equilibrium bedform morphology. Additionally, the system reproduces the scaling of topographic roughness through space and time seen in natural fluvial dunes (Jerolmack and Mohrig 2005a) and aeolian dunes (Swanson et al. 2017). Because the bedform surface modeling strategy adopted by Jerolmack and Mohrig (2005a) and Swanson et al. (2017) has proven to provide a robust characterization of dune autogenic processes, it is an ideal tool to explore the coevolution of bedform topography and stratigraphy under various environmental forcings.

The original bedform modeling framework presented by Jerolmack and Mohrig (2005a) is here modified to evolve both topography and stratigraphy driven by two allogenic boundary conditions: sediment transport capacity as determined by boundary shear stress and bed aggradation rate. In this paper, a total of three scenarios are explored, each composed of three depositions, each of which is herein defined as an individual episode of deposition of duration $2.5 \times 10^4$ times steps ($\Delta t$), yielding a total simulation time of $7.5 \times 10^4$ timesteps per model scenario. In all scenarios, the timestep is held constant ($\Delta t = 1$), and the subscription $j$ is used to indicate the $j$th timestep. The first scenario simulates bedform growth from a roughened sandy bed with steady sediment transport capacity driven by a steady ambient (global) shear stress term, $\tau_{bg} = 0.3$, and zero bed aggradation $r_{ag} = 0$ per timestep. The second scenario includes depositions composed of a single period of sinusoidal variation in ambient shear stress with a steady bed aggradation rate of $r_{ag} = 5 \times 10^{-3}$ per timestep. In scenario 2, over a single deposition, $\tau_{bg}$ varies between a maximum value of 0.3 and a minimum value of 0.24 ($\Delta \tau_{bg} = 0.20$). This environmental forcing is chosen to conceptualize an aeolian system subject to relative increases and decreases in wind strength over climate oscillations resulting in waxing and waning sediment transport capacity (Kocurek 1999). To complete the numerical experiment, a third scenario considers steady sediment transport capacity ($\tau_{bg} = 0.2$) and constant bed aggradation of, $r_{ag} = 5 \times 10^{-3}$ per timestep. Unless otherwise indicated, in the following expressions all simulations use the same set of parameters. Additionally, a comprehensive table of all parameters, their physical meaning, and units is provided in the supplementary materials.

During each simulation, conservation of sediment is approximated by a finite-volume method. The procedure adopted to solve the equation for bedform surface evolution closely follows the original model presented by Jerolmack and Mohrig (2005a). Boundary shear stress scales with the aspect ratio of a bedform, $\tau_{bg} \propto h^{1/3}$, where $h$ and $\lambda$ are the height and wavelength of a bedform (Jackson and Hunt 1975; Kroy et al. 2002). Therefore, any increase in bedform crest height or surface slope will create a proportional increase in boundary shear stress over the stoss slope of the bedform. This fundamental behavior of total boundary shear stress $\tau_b$ (subscript $i$ indicates the $i$th node) increasing with flow blockage and shoaling is approximated for each timestep ($j$) as a generic expansion of topography (Jerolmack and Mohrig 2005a),

$$
\tau_{bi} = \tau_{bo} \left(1 + A (\eta_i - \eta_{oi}) + B \frac{(\eta_i - \eta_{oi-1})}{\Delta x} \right) \tag{1}
$$

where flow blockage and shoaling are described by coefficients $A = 0.1$ and $B = 3$, respectively (Jerolmack and Mohrig 2005a), $\Delta x$ is the spacing between nodes, and $\eta_{oi}$ is the cumulative sediment added during previous $i$ timesteps. $\eta_{oi}$ is removed from the calculation of boundary shear stress as otherwise during scenarios 2 and 3; each timestep would cause global increases in boundary shear stress. The practice of expressing boundary shear stress as a function of topographic height and slope is deeply rooted in early studies of fluid motion over bedform topography (Exner 1925). The relationship between boundary shear stress and topography approximates the along-stoss-slope trend in boundary shear stress derived from sediment flux over a stoss slope of an aeolian dune observed by Lancaster et al. (1996). To approximate the transport conditions in a lee shadow zone, nodes with boundary shear stress less than zero are set to zero,

$$
\tau_{bi} = \begin{cases} 
\tau_{bb} & \tau_{bi} > 0 \\
0 & \tau_{bi} \leq 0 
\end{cases} \tag{2}
$$

because boundary shear stress along a lee slope is always computed as negative (Jerolmack and Mohrig 2005a). Closely following the bedform modeling strategies of Diniega (2010), Jerolmack and Mohrig (2005a), and Hersen (2004), lee slopes that exceed a threshold angle, $\theta_l = 32^\circ$, relax via down-wind calculated diffusion, written as

$$
q_{sl} = \begin{cases} 
E \left(\frac{(\eta_i - \eta_{oi-1})}{\Delta x} \right)^2 - \tan \theta_l \left(\frac{(\eta_i - \eta_{oi-1})}{\Delta x} \right) & \text{if } \eta_i - \eta_{oi-1} \geq 0 \\
0 & \text{if } \eta_i - \eta_{oi-1} < 0 
\end{cases} \tag{3}
$$

A large avalanching coefficient $E = 20$ is chosen so that lee slopes relax by avalanche fluxes, $q_{sl}$, to an angle of repose in approximately a single
time step (Jerolmack and Mohrig 2005a; Diniega 2010). Lee slopes below the threshold angle do not trigger the calculation of avalanche fluxes (Eq. 3). The magnitude of all fluxes, \( q_{si} \), is then computed for each node as \( q_{si} = m r_{ai}^n + q_{se} \), using the power-law relationship of Meyer-Peter and Muller (1948) with a coefficient \( m = 1 \) and exponent \( n = 3/2 \) (Jerolmack and Mohrig 2005a). The chosen power-law relationship is comparable to the ballistic formula of Bagnold (1941) and was chosen to represent saturated sediment flux over bedform topography (Kroy et al. 2002). The elevation change at each node is calculated by

\[
\Delta h_i = \frac{-\Delta t}{(1-p)\Delta x} (q_{si} - q_{se,i-1}) + \frac{\Delta t D}{(\Delta x)^2} (\eta_{i+1} + \eta_{i-1} - 2\eta_i) + r_{ai} \tag{4}
\]

which combines a first-order upwind finite difference of sediment flux \( q_{si} \), accounting for porosity, \( p = 0.4 \), with a first-order approximation of topographic diffusion with diffusivity \( D = 0.2 \). Topographic diffusion is superimposed for enhanced numerical stability (Press et al. 1996; Jerolmack and Mohrig 2005a).

**Model Domain, Boundary, and Initial Conditions**

The 1D modeling domain is composed of 1001 nodes with uniform spacing \( \Delta x = 10 \). A periodic boundary condition is formed by allowing the first and last nodes to exchange sediment as if contiguous. This boundary condition allows bedforms to repeatedly cycle through the domain, which represents the temporal evolution of dunes in an interior part of a dry sand sea, where sediment accumulation and eventual preservation are likely to occur in natural settings. All runs are initialized with a roughened bed of low-amplitude random topography uniformly distributed about a mean of 0.1. Different initial conditions yield different topographic fields for the same model scenario, and therefore create different stratigraphy. To sample model dependence on the initial bed topography, an ensemble of model results is obtained for each scenario by running a set of 12 realizations of initial bed topography, resulting in a total of 36 runs. Ensembles of results allow signal averaging, which is a time-domain signal processing technique that strengthens signals relative to noise, which, in this case, noise could be conceptualized as the model dependence on initial bed topography. Larger ensembles of initial conditions and model runs were explored; however, a set of 12 initial conditions was found to adequately sample the variability in topography and stratigraphy that arises from specific initial bed configurations. However, for consistency, all presented stratigraphic sections are generated using the sixth initial condition of bedtopography from the set of twelve initial conditions used to generate ensembles of simulation results.

**Topographic and Stratigraphic Post-Processing**

Post-processing of both topography and stratigraphy is performed at specified time intervals. To capture rapid changes during early simulation time, model time steps are post-processed every 100\( \Delta t \) for \( t < 5000\Delta t \). Afterward, to reduce computational cost, this interval is increased to 700\( \Delta t \), resulting in a total of 200 samples per run. For each sample, consecutive pairs of dune crest and trough elevation are identified and differenced to calculate dune height, \( h \); likewise, dune crest positions are differenced to calculate dune wavelength, \( \lambda \). A stratigraphic section is then constructed using all timesteps of dune topography up to the time step of interest. Afterward, the elevation of all erosional surfaces, otherwise known as bounding surfaces, are identified in the section. For each sample, the cumulative number of dunes that have passed each node and the number of bounding surfaces above each node are counted. Successive bounding-surface elevations are differenced vertically to calculate the thickness of sets of cross-strata, which are herein referred to as set and set thickness, \( l_{si} \). Any set with \( l_{si} < 0.01 \) is discounted, because it is unlikely that such a thin unit will be identified as an independent set of cross-strata in actual sedimentary deposits. Additionally, in analyses that relate distributions of dune height to distributions of \( l_{si} \), dune heights are filtered to include only events that occur after the creation of the earliest bounding surface in each stratigraphic section.

To facilitate comparison with physical systems, the vertical, horizontal, and time scales of modeled stratigraphic sections are nondimensionalized by the dynamic equilibrium values of dune height, wavelength, and deposode period, respectively, creating nondimensionalized elevation, \( \eta_i = \frac{\eta_i}{h_{eq}} \) horizontal distance, \( x = \frac{x}{\lambda_{eq}} \) and simulation time, \( t' = \frac{t}{2.5 \times 10^5 \Delta t} \). For each run, \( h_{eq} \) and \( \lambda_{eq} \) are found by a nonlinear least-squares fitting of an exponential-growth-to-saturation model, \( \chi(t') = s_{eq}(1 - e^{-t' / \tau}) \), where the variables \( s \) and \( s_{eq} \) represent the time-series and dynamic-equilibrium values of the morphological scale of interest, respectively (Baas 1994). Deposode period is obtained from the wavelength of \( \tau \) in scenario 2. This practice places model results in a conceptual reference frame of equilibrium dune morphology and deposode duration.

**RESULTS**

**Dynamic Dune Scales**

During early simulation time, scenarios show similar temporal changes in spatially averaged and ensemble-averaged dune height, \( \bar{h} \), and dune wavelength, \( \bar{\lambda} \) (Fig. 1A, B). Throughout the simulation, \( \bar{h} \) is measured as the vertical distance between the spatially averaged and ensemble-averaged dune crest and trough elevations (Fig. 1C), and \( \bar{\lambda} \) is dune crest-to-crest horizontal distance. Starting from the initial condition, each simulation exhibits a brief duration of very slow change in bed configuration (Fig. 1A, B). This corresponds to low values of sediment flux and boundary shear stress computed along initial low-lying topography (Eq. 1). Gradually, low-amplitude bedforms self-organize from initial bed roughness, slowly increase in size, and begin to coalesce. After initial growth and coalescence, \( \bar{h} \) and \( \bar{\lambda} \) rapidly increase, then saturate, reaching a dynamic equilibrium by \( t' \approx 0.2 \) (Fig. 1A, B). After reaching dynamic equilibrium, \( \bar{h} \) and \( \bar{\lambda} \) begin to respond to the set of environmental forcings unique to each scenario. For example, in scenarios 1 and 3, which do not include time varying values of \( \sigma_{ai} \), dunes exhibit exponential growth, then saturation to an equilibrium morphology—a characteristic response of bedforms evolving under steady unidirectional flows (Baas 1994) (Fig. 1A). In contrast, after initial growth, dunes in scenario 2 respond to sinusoidal variation in \( \sigma_{ai} \) with a similar sinusoidal oscillation about an equilibrium value of \( h \) (Fig. 1A). Notably, the equilibrium value of \( \bar{\lambda} \) is significantly larger in scenario 2, but relatively insensitive to fluctuations in \( \sigma_{ai} \) (Fig. 1B). In strong contrast to mean dune scales, for each scenario, the time series of the standard deviation of ensemble dune height, \( \sigma_{ai} \), is nearly identical throughout all simulation time. At first, \( \sigma_{ai} \) rapidly increases to a maximum value at approximately \( t' \approx 0.1 \). Afterward, at first, \( \sigma_{ai} \) decreases at a rapid rate, then at approximately \( t' \approx 0.2 \), the slope of \( \sigma_{ai} \) decreases slowly for the rest of simulation time.

**Topographic and Stratigraphic Co-Evolution**

**Early Simulation Time.**—As with early changes in mean dune scales, stratigraphic sections during the initial stages of bedform growth and merger in each scenario are nearly identical. Within the first hundred timesteps of a deposode, bedforms spontaneously emerge from the rough sandy bed and develop internal stratification. At this time, dunes exhibit significant morphological variability, underscored by differences in height, wavelength, and the crest and trough elevations (Figs. 1D, 2A). Due to this variability, these early dunes differ in celerity, allowing dunes to collide and merge. In these early timesteps, individual dunes increase in size by scouring into bed material below initial bed elevation (dunes on left and right side, Fig. 2), and from dune merger (left side, Fig. 2B, C).
Occasionally, dune troughs may scour through the stoss surfaces of downwind dunes, creating new truncation surfaces, and new cross-sets with significant thickness, which can approach the height of individual dunes. This newly created stratigraphy is laterally discontinuous, but in places it contains a substantial record of early dune self-organization. For example, portions of the domain document the passage of multiple dunes, as indicated by multiple, vertically stacked bounding surfaces (middle region, Fig. 2C).

After initial dune coalescence, continued dune growth is sustained by rapid scouring of bed materials (Fig. 1C) and consequent cannibalization of the earliest synthetic rock record in all scenarios (Fig. 3A). Across the domain, dune troughs descend nonuniformly, creating self-organized groups of dunes with higher troughs and groups with lower troughs (arrows, Fig. 3A). Further dune growth ceases as troughs descend significantly below mean bed elevation, and \( t_o \) becomes vanishingly small, which leaves substantial spatial variation in scour depth and individual-dune celerity throughout the domain. Hereafter a dune group is defined as a single wavelength of the sinusoidal variation in scour depth shown in Figure 4. Afterward, spatial variation in scour depth propagates in the transport direction at a group celerity. Trough motion imparted by group celerity occurs in the same direction as individual dune motion. The motion of dune troughs within dune groups is achieved by the same morphodynamic feedback (Equations 1 through 4), simply manifesting as a long-wavelength (\( \sim 5 \lambda_{eq} \) to \( 10 \lambda_{eq} \), low-amplitude (\( \sim 0.1 h_{eq} \) to \( 0.2 h_{eq} \)) sinusoidal variation in dune scour depth (dune group, Fig. 4). Dune-group motion is accomplished by way of routing sediments through individual dunes. Within a dune group, upward dunes are arranged in increasing elevation in the down-transport direction (Fig. 4). Due to these consecutive increases in elevation, each dune experiences more boundary shear stress (Eq. 1, \( A \) coefficient) and scours more vigorously compared to its upwind neighbor. This ultimately causes deflation of the upwind part of the dune group and routes an excess of sediment through ascending dunes toward the leeward part of the dune group. Along the leeward segment, each consecutive dune is slightly lower in elevation (Fig. 4). Because of this, each downwind dune receives more sediment than it can transport over its stoss slope. By conservation, this causes dunes on the leeward part of the dune group to ascend. Therefore, through the paired motion of scouring upwind dunes and ascending leeward dunes, the dune group moves forward; this motion is clearly visible in animations, see Supplemental Materials. Therefore, after the formation of dune groups (Fig. 3A), lower dune troughs ascend (Fig. 3B), and begin to create a substantial record (arrows, Fig. 3C). The lowest erosional surface shown in Figure 4C represents the farthest descent of dune troughs and the lowest part of initial dune groups. This scour depth is never revisited and represents the lowest bounding surface in all model scenarios.

**Long-Term Topographic and Stratigraphic Co-Evolution.**—In each scenario, immediately after group formation, the passage of individual dune troughs within groups create co-sets (Figs. 4C, 5). Within a dune group, dune troughs at the lowest part of a dune group are slowly carried upward as the dune group moves forward, because group celerity is faster than individual dune celerity. This combined motion of dune troughs in dune groups produces individual sets that start near zero thickness near the bottom of a co-set, and typically show upward increases in bounding-surface slope and thickness until ending in a modern dune (annotated dune group, Fig. 4), or bounding surface. As dunes continue to self-organize, variability in dune-trough elevation decays with simulation time, as indicated by decreases in \( \sigma_{h} \) (Fig. 1D). In all sections, this decay in dune disorder is recorded in the architecture of the synthetic sections as an upward transition from co-sets composed of a few, large sets separated by steep bounding surfaces toward subhorizontal bounding surfaces, and more numerous, thinner sets packaged within later co-sets (Figs. 4, 5, 6, 7). However, the exact way this autogenic signal of self-organization is presented in synthetic stratigraphy is unique to each scenario. Many of the observations in this section are based on videos that show the longer-term (\( t' = 0.2 \) to 3) co-evolution of dune topography and stratigraphy. Although only the final synthetic sections are shown below, video files of each scenario are available, see Supplemental Materials.

The final stratigraphic section of scenario 1 shows architecture generated from allogenic boundary conditions of steady transport capacity, zero net bed aggradation, and autogenic self-organization of dunes within dune groups (Fig. 5). During deposition, the deepest dune trough in each group varies between groups, and changes with simulation time due to ongoing self-organization. This spatially and temporally variable group-scour depth causes frequent lateral truncation of group-deposited co-sets that dominate the architecture present in Figure 6. Although highly compartmentalized in laterally truncated co-sets, generally, set thickness and bounding-surface slope decrease, and number of sets per co-set
increases upwards through the section (Fig. 5). A video showing the co-evolution of dune topography and stratigraphy is available, see Supplemental Materials (scenarioOne.mp4).

Despite the allogenic boundary conditions of steady bed aggradation and substantial sinusoidal fluctuations in ambient boundary shear stress, all records of initial dune growth and motion are cannibalized in the final synthetic section of Scenario 2 (Fig. 6). However, as \( \tau_a \) increases during periodic fluctuations, sediment transport rates increase, which allows dune troughs to scour deeper, driving an increase in \( h \) (Fig. 1A, C). During increases in \( \tau_a \), the scouring action of dune groups outpaces trough elevation gain associated with the overall aggradation of the bed (Fig. 1C). The rhythmic motion of dune trough descent and ascent with waxing and waning \( \tau_a \) drives amalgamation of sets into nearly laterally continuous bounding surfaces with significant undulations in elevation (Fig. 6). Similarly, the ascent and descent of dune troughs form large-scale undulations in the thickness of both individual sets and co-sets (Fig. 6).

Although comingled with changes imparted by fluctuations in transport capacity, individual set thicknesses and bounding-surface slopes generally decrease, and the number of sets in a co-set generally increase up-section. A video showing the coevolution of dune topography and stratigraphy is available, see Supplemental Materials (scenarioTwo.mp4).

In scenario 3, where sediment transport capacity and bed aggradation are constant, the earliest records are cannibalized by dune self-organization and group formation (Fig. 7). However, in strong contrast to sections generated by scenarios 1 and 2, group-deposited co-sets appear vastly more tabular, and extend across the entire section. In Figure 8, the upward transition from a few, thick, highly inclined sets in each co-set toward co-sets containing numerous, thin, subhorizontally inclined sets generated by the ongoing self-organization of dune troughs within dune groups is exceptionally clear. A video showing the coevolution of dune topography and stratigraphy is available, see Supplemental Materials (scenarioThree.mp4).

Fig. 2.—Sequential stratigraphic sections sampled every 0.08 \( t' \) from scenario 1, initial condition 6. A) Internal stratification formed from initial dune motion. B) Center dunes migrate over each other, forming a significant early stratigraphic record, while dunes to the left and right scour into previously unscoured bed material. C) Rightmost dunes merge, while centered dunes maintain a significant stratigraphic record. Synthetic stratigraphy is color-mapped by time of deposition, \( t' \). Vertical exaggeration = 25x.
Fraction of Dunes Preserved as Sets.—In the first hundred timesteps of a deposode \((t' = 0\) to \(0.05\)), initial topography is rapidly worked into low-lying, disorganized bedforms (Fig. 1). Their forward motion causes a notable increase in the average number of dunes that have visited each node, \(n_d\) (Figs. 2, 8A, B), and average number of sets at each node, \(n_s\) (Figs. 2, 8C, D). Likewise, the fraction of dunes preserved as sets, \(j = n_s n_j^{-1}\), increases to a local maximum (Fig. 8E). Immediately afterward, precipitous declines in \(n_d\), \(n_s\), and \(j\) (Fig. 8B, C, E) coincide with continued dune growth (Fig. 1A), dune merging (Fig. 2), and amalgamation of sets (Fig. 3A). Next, by \(t' \approx 0.1\), the forward motion of non-uniform dune-trough elevations in dune groups begins to create new sets (Fig. 4B, C), which correlate to rapid increases in \(n_d\), \(n_s\), and \(j\) (Fig. 8A, C, E). From there on, in each scenario, \(n_s\) increases monotonically and exhibits a gradual, but continuous, increase in slope (Fig. 8D). For the zero-aggradation case, scenario 1, \(n_s\) increases at a nearly constant rate (Fig. 8B). Conversely, set creation by forward motion of dune groups shows significant variability between scenarios and over the rest of each scenario (Fig. 8D). For the zero-aggradation case, scenario 1, \(n_s\) increases monotonically with a gradually decreasing slope (Fig. 8D). In the case of scenario 2, \(n_s\) oscillates about a generally increasing trend. However, in scenario 3, which includes constant aggradation, \(n_s\) increases monotonically and exhibits a gradual, but continuous, increase in slope (Fig. 8D). In all scenarios, \(j\) increases rapidly until \(t' \approx 0.2\), beyond which \(j\) shows marked differences. During the rest of scenario 1, \(j\) decays slowly, suggesting that fewer dunes are preserved with increasing simulation time (Fig. 8E). Conversely, in scenario 3, \(j\) decreases at first, then past one deposode, begins to increase slowly for the rest of simulation time (Fig. 8E). Despite periodic perturbation by waxing and waning \(r_s\) in scenario 2, decay of dune trough elevation variability combined with constant aggradation allows more, but thinner, sets to be preserved by pulsing movements of dune groups (Fig. 6), this type of motion gives \(j\) strong oscillations superimposed on a trend similar to that of scenario 3 (Fig. 8E).

Distributions of Dune Height and Set Thickness.—During each simulation, topography and stratigraphy are sampled to obtain growing populations of dune height and set thickness. The distribution of each node, \(n_d\) (Fig. 8A, B), and average number of sets at each node, \(n_s\) (Figs. 2, 8C, D). Likewise, the fraction of dunes preserved as sets, \(j = n_s n_j^{-1}\), increases to a local maximum (Fig. 8E). Immediately afterward, precipitous declines in \(n_d\), \(n_s\), and \(j\) (Fig. 8B, C, E) coincide with continued dune growth (Fig. 1A), dune merging (Fig. 2), and amalgamation of sets (Fig. 3A). Next, by \(t' \approx 0.1\), the forward motion of non-uniform dune-trough elevations in dune groups begins to create new sets (Fig. 4B, C), which correlate to rapid increases in \(n_d\), \(n_s\), and \(j\) (Fig. 8A, C, E). From there on, in each scenario, \(n_s\) increases at a nearly constant rate (Fig. 8B). Conversely, set creation by forward motion of dune groups shows significant variability between scenarios and over the rest of each scenario (Fig. 8D). For the zero-aggradation case, scenario 1, \(n_s\) increases monotonically with a gradually decreasing slope (Fig. 8D). In the case of scenario 2, \(n_s\) oscillates about a generally increasing trend. However, in scenario 3, which includes constant aggradation, \(n_s\) increases monotonically and exhibits a gradual, but continuous, increase in slope (Fig. 8D). In all scenarios, \(j\) increases rapidly until \(t' \approx 0.2\), beyond which \(j\) shows marked differences. During the rest of scenario 1, \(j\) decays slowly, suggesting that fewer dunes are preserved with increasing simulation time (Fig. 8E). Conversely, in scenario 3, \(j\) decreases at first, then past one deposode, begins to increase slowly for the rest of simulation time (Fig. 8E). Despite periodic perturbation by waxing and waning \(r_s\) in scenario 2, decay of dune trough elevation variability combined with constant aggradation allows more, but thinner, sets to be preserved by pulsing movements of dune groups (Fig. 6), this type of motion gives \(j\) strong oscillations superimposed on a trend similar to that of scenario 3 (Fig. 8E).
sample population of dune heights is described using the coefficient of variation, $c_v = \sigma_h h_c^{-1}$ (Fig. 9A), where $\sigma_h$ and $\mu_h$ are the standard deviation (Fig. 1D) and mean values of dune height (Fig. 2A) calculated using estimates of shape and rate parameters from maximum likelihood gamma distribution fits. Immediately after the first timestep, $c_v$ rapidly increases to its maximum value at $t^* \approx 0.04$. This occurs in all scenarios and corresponds to initial working of a rough bed into the earliest, least organized dunes, as indicated by high values of $c_v$ (Figs. 3, 10A). Afterward, during rapid dune growth, $c_v$ decreases quickly until individual dunes are organized into groups, at $t^* \approx 0.2$ (Fig. 9A). After this precipitous decrease, generally, $c_v$ gradually decreases for the remaining simulation time (Fig. 9A). Notably, however, scenario 2 shows faint oscillation, and maintains slightly larger values of $c_v$ compared to scenarios 1 and 3 (Fig. 9A). Similarly, mean set thickness, $\mu_h$, estimated from maximum-likelihood exponential distribution fits, shows a dramatic increase, attaining a maximum value just after initial formation of dune groups at $t^* \approx 0.1$ (Fig. 9B). Afterward, $\mu_h$ generally declines with simulation time. However, during later simulation time, scenario 2 and 3 tend to have larger values of $\mu_h$, which is attributable to constant aggradation. However, scenario 2 maintains the largest values of $\mu_h$, which oscillate in response to changes in dune-trough elevation driven by fluctuations in $\tau_u$ (Fig. 9B). At first, the preservation ratio, $\omega = \mu_h / \mu_l$, (Paola and Borgman 1991), rapidly decreases during very early simulation time (Fig. 8C), then, corresponding to the initial formation of dune groups, $\omega$ reaches a local maximum at $t^* \approx 0.1$ (Fig. 9C). Afterward, $\omega$ rapidly decays as ongoing dune group motion begins to truncate the largest sets at the lowest part of every scenario (Fig. 3C). Similar to $\kappa$, after $t^* \approx 0.2$, the trajectory of $\omega$ is different from each scenario. In scenario 1 further changes in $\omega$ are very similar to $\kappa$ (Fig. 9D), where ongoing decay of dune-trough elevation variability creates thinner sets upsection (Fig. 5). However, despite constant aggradation, ongoing decreases in dune-trough elevation variability cause slower, but monotonically decreasing, $\omega$ for the rest of scenario 3. Similarly, scenario 2 shows a general decrease in $\omega$, with superimposed oscillations, which correspond to subtle fluctuations in set thickness due the punctuated scour and deposition from dune groups, driven by sinusoidal fluctuations in $\tau_u$.

**DISCUSSION**

This numerical experiment clearly shows two distinct periods of topographic and stratigraphic coevolution. In all examined scenarios, dune scale, motion, stratigraphy, and preservation are strongly similar during the first quarter of a deposode. Afterward, the characteristic rapid changes in dune morphology and motion slow down, and the ongoing morphodynamic processes operating in a dune field become sensitive to changes in environmental forcing. To help identify systematic changes during the coevolution of bedform topography and stratigraphy, the stochastic theory of Paola and Borgman (1991) is applied as a theoretical benchmark for dune preservation in each scenario.

**Benchmarking Dune Preservation**

Paola and Borgman (1991) envisioned set creation to arise from the passage of a train of bedforms scouring to random depths, working and reworking sediment with zero net bed aggradation. In this framework, the preservation ratio, $\omega$, is related to the coefficient of variation of bedform height, $\omega = 1.645 c_v^2 / \varepsilon_e$, where the reference level, $\varepsilon_e$, is a cutoff that segments the distribution of dune height into set-creating versus non-set-creating parts. In the case of Paola and Borgman (1991), $\varepsilon_e$ is set equal to 2, allowing only scour below mean bed elevation to create sets. While the modeled scenarios here are markedly different from this theoretical system, the application of this stochastic framework provides a benchmark for dune preservation in the case where dune-trough elevations are randomly...
FIG. 6.—Stratigraphic section from scenario 2, initial condition 6. Synthetic stratigraphy is color-mapped by the time of deposition, $t$. Plot vertical exaggeration $= 100$; however, for visualization, topography has been downscaled by a factor of 0.05.
FIG. 7.—Stratigraphic section from scenario 3, initial condition 6. Synthetic stratigraphy is color-mapped by the time of deposition, \( t^* \). Plot vertical exaggeration = 100; however, for visualization, topography has been downscaled by a factor of 0.05.
distributed in space and time, and is referred to here as the PB theory (Fig. 10). Therefore, a reasonable supposition would be that any significant departure from this expected relationship may reflect systematic changes in bedform morphology and/or preservation, attributable to autogenic dune processes such as self-organization and/or geomorphic responses to allogenic boundary conditions.

Within the first few hundred timesteps of a deposode, the forward motion of small, disorganized, and rapidly deforming dunes creates populations of dune height and set thickness (Figs. 2, 9C) that are in reasonable agreement with PB theory ($t^* \leq 0.05$, Fig. 10). Immediately afterward, increases in scour depth (Fig. 1C), and ongoing dune merger (decline in $n_d$, Fig. 8A) drive increases in both $\mu_d$ and $\sigma_h$ (Fig. 1). However, increases in $\mu_d$ are unevenly accommodated by changes in both dune-crest and dune trough elevation (Fig. 1C), and outpace simultaneous increases in $\mu_d$ (Fig. 9B), which drives both a precipitous decline in $\omega$ (Fig. 9C) and significant excursion from PB theory ($t^* \leq 0.1$, Fig. 10). During this period, the ongoing descent of dune troughs during dune-group self-organization locally amalgamates sets into bounding surfaces in an asynchronous manner, evidenced as $\kappa$ never returns to zero (Fig. 8E). This duration of substantial set amalgamation effectively clears the stratigraphic memory, shreds any systematic relationship between dune topography and stratigraphy, and returns all scenarios toward temporary agreement with PB theory by $t^*/C^3 \geq 0$ (Fig. 10). Afterward, all scenarios largely run subparallel to PB theory, with the exception of a brief increase in $\omega$, at $t^* \approx 0.2$, which corresponds to a highly ephemeral, yet robust, record created by the passage of the first dune group (Fig. 3C), which is partially cannibalized by the passage of later groups.
PB theory does not predict temporal changes between moments of dune topography and stratigraphy, yet, aside from early periods of dune growth and group formation ($t^*>0.1$), preservation trends largely run subparallel to PB theory ($t^*>0.1$, Fig. 10). This offset trend is interpreted as a self-organization signal, which arises from ongoing homogenization of dune-trench elevations (Fig. 9A) and consequential decreases in preservation (Fig. 9C). However, superimposed on this generalized trend are subtle changes in $\omega$ and $c_v$, which correspond to each set of allogenic boundary conditions. For example, the unsteady transport capacity in scenario 2 corresponds to fluctuations in $\omega$ (Figs. 9C, 10). Furthermore, scenario 2 ends at the highest values of $\omega$ and $c_v$, suggesting that variability in environmental forcing helps maintain topographic variability, and consequently, bolsters preservation. Past $t^*=0.2$, the rest of scenario 1 plots significantly below scenario 3 (Fig. 10), which is directly attributable to steady aggradation in scenario 3.

**Path Forward and Error**

The co-evolution of dune topography and stratigraphy produced by this model is unlikely to exactly match any specific physical analog. However, the process-based interpretations of architectural elements in the experimental sections define five working hypotheses that may help guide the interpretation of any succession of aeolian strata. Firstly, dune growth is a spontaneous autogenic process that cannibalizes records of the youngest dunes, and therefore, shreds any early environmental record of a system. This result may help to support recent field interpretations of cannibalization of early dune deposits in the Entrada Sandstone (Kocurek and Day 2018). However, a potentially useful corollary to this hypothesis is that confirmed preservation of early dune accumulations may be diagnostic of an external control on dune preservation, such as water-table rise, precipitation that would shutoff aeolian transport, or early dune deposition in antecedent topographic lows (Cardenas et al., this issue). Secondly, self-organization into groups of dunes, and subsequent group-based deposition of co-sets, is an important process in the creation and preservation of aeolian stratigraphy. This hypothesis is comparable to preservation mechanisms proposed by previous studies which have focused on set-scale preservation occurring in nested scales of fluvial morphological elements including bars, channels, and channel belts (Herbert and Alexander 2018; Paola et al. 2018; Miall 2015; Reesink et al. 2015). Additionally, the forward motion of dunes in dune groups drives modest dune climb, and the scouring action of dune group troughs may create outcrop-scale truncation surfaces, which, in this experiment, arise from autogenic processes, but would typically be recognized as parasequences and super-surfaces, and therefore possibly thought to originate from allogenic boundary conditions. For example, while large-scale bounding surfaces in the Page Sandstone are interpreted to have arisen from increases in base-level (Cardenas et al., this issue), bounding surfaces of similar scale may be generated by the scour and fill associated with the passage of dune groups in dry systems.

The third working hypothesis is that a dune field becomes increasingly sensitive to its set of allogenic boundary conditions as dune self-organization rates slow over time. This allows allogenic and autogenic signals to comingle and enter the aeolian rock record as the entire field matures. Temporal changes in preservation due to self-organization provide a plausible pathway for future workers to deconvolve signals arising from dune autogenic behavior from changes in the imposed allogenic boundary conditions. For example, Cardenas et al. (this issue) observed that set-thickness variability within a well-established wet aeolian system (Entrada Sandstone) is measurably less than what is expected from random scouring in a dry system such as the Page Sandstone. Finally, all numerical scenarios of set variability plot below that of the PB theory (Fig. 10), suggesting that self-organization of dunes within groups reduces preservation compared to a truly random bedform system. Although PB theory has been combined with a geometric model (Allen 1970) to account for bedform climb (Bridge and Best 1997), the chosen reference level, $e$, has been carefully modified to match experimental topography and resulting stratification (Leclair and Bridge 2001); further work could explore bedform preservation under the influence of self-organization. Regardless of environment or planet, self-organization of bedforms into fewer, larger forms during bedform pattern formation is a ubiquitous process (Day and Kocurek 2018), and therefore may have profound implications for the reconstruction of bedform topography from cross-stratified deposits.

The succession of temporal changes in bedform morphology and preservation documented in this numerical experiment may be conceptually transferable to spatial changes along the margins of a sand sea. For example, a recent study documented spatiotemporal changes in bedform morphology, and spatial changes in recent deposit architecture from the youngest protodunes to the first mature dune downwind along an upwind margin of a sand sea (Phillips et al. 2019). In their study, they found that bedform deposition did not occur until the third downwind protodune and development of the first mature dune occurred by many mechanisms, including protodune growth, interaction, and merging. These observations are roughly congruent with early simulation behavior in all modeled scenarios where initial, low-amplitude, long-wavelength bedforms climb over each other, creating a significant but spatially discontinuous sedimentary record (Fig. 2). Although beyond the scope of this initial work, future applications of the bedform strata-formation model could choose to explore dune growth and preservation from other allogenic boundary conditions, such as an upwind margin of a sand sea where detailed studies of modern systems could benchmark model performance.

These experimental results and interpretations are the products of mass-conserving numerical approximations for bedform topography evolution (Jerolmack and Mohrig 2005a). Because of the heuristic nature of this model, there is no analytical solution in which to validate or otherwise constrain error that arises from the numerical treatment. However, the bedform surface model recreates the scaling behavior of natural fluvial and aeolian bedform topography and reproduces fundamental bedform morphodynamic behavior, such as development of self-organized patterns through bedform growth and interactions (Jerolmack and Mohrig 2005a; Swanson et al. 2017). However, this bedform modeling strategy cannot...
simulate dune behavior associated with sediment transfer between dunes via sediment entrainment in a turbulent wake (Mohrig and Smith 1996; Swanson et al. 2018). Therefore, application and perception of this exploratory model should be tempered by its simplicity, and the results of this study warrant further exploration with theoretical models, physical models, natural dune fields, and ancient aeolian rock records (Cardenas et al., this issue).

CONCLUSIONS

A one-dimensional morphodynamic model of bedform topography is adapted to create two-dimensional synthetic sections of aeolian stratigraphy arising from three different sets of allogenic boundary conditions: 1) steady transport capacity, 2) steady bed aggradation and variable transport capacity, and 3) steady transport capacity and bed aggradation. In each scenario, the initial roughened sandy bed is quickly worked into a field of small, disorganized dunes. The forward motion of these initial dunes generates a significant record exclusively containing autogenic signals that arise from early dune growth, deformation, merger, or generically described as dune self-organization. However, despite steady bed aggradation in scenarios 2 and 3, continued dune growth and self-organization into dune groups thins all records of early dunes in all scenarios. Shortly after group formation, dune scales reach dynamic equilibrium, and rates of self-organization slow down. Forward motion of individual troughs in dune groups create and truncate co-sets of cross-strata, quickly forming a second, significantly more robust stratigraphic record, which preserves a comingling of signals sourced from slowed self-organization and each scenario’s specific set of allogenic boundary conditions. Interestingly, in all scenarios, preserved signals of self-organization are sourced from ongoing homogenization of dune-trench elevations, and consequential decreases in preservation, which is found to generally follow to a well-known stochastic theory that relates moments of bedform topography to stratigraphy.

The numerical experiments here do not explore different wind regimes, sediment-supply limitations, bedform morphologies, or the types of bedform interactions that are possible to simulate using the bedform surface modeling approach (Jerolmack and Mohrig 2005a; Swanson et al. 2017). To encourage further exploration of the interplay between autogenic processes and allogenic forcings in aeolian stratigraphy, the authors would like to thank Mackenzie Day, an anonymous reviewer, John Gillies, and Peter Burgess for their constructive reviews and edits. Furthermore, John Southard provided copy edits which dramatically improved the quality of this document. This work is an extension of a dissertation chapter which received vastly helpful comments and edits from Wonsuck Kim, Joel Johnson, and Paola Passalacqua, and was partially supported by Shell International Exploration & Production Inc. Additionally, the Shell Center for Sustainability at Rice University supported a part of this work. However, this work does not reflect the views of Shell International Exploration & Production Inc.

REFERENCES


SUPPLEMENTAL MATERIAL

Supplemental files are available from JSR’s Data Archive.


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