Reverse migration of lithofacies boundaries and shoreline in response to sea-level rise

Carolina Baumanis and Wonsuck Kim
Department of Geological Sciences, Jackson School of Geosciences, University of Texas, Austin, TX, USA

ABSTRACT

The migration of the lithofacies boundaries preserved in the sedimentary record is key to interpreting changes in depositional environments. Grain size is one of the most recognizable physical characteristics of lithofacies. The advance and retreat of grain-size breaks, as a proxy for lithofacies boundaries (e.g. gravel–sand transition), is commonly attributed to variations in external controls (e.g. climate, sea level and tectonic subsidence). While most models of fluviodeltaic systems focus on predicting the response of the shoreline to these forcings, none have thoroughly incorporated the migration of grain-size transitions (GST) that coevolve with the shoreline. We present a numerical delta evolution model that treats both the shoreline and GST as moving boundaries to provide quantitative understanding of the dynamic interaction between the downstream boundary (shoreline) and the upstream lithofacies boundaries (GSTs) of the fluviodeltaic system under relative sea-level rise. We tested a range of relative sea-level rise rates in the model. The shoreline and GST gradually reduced their progradation rates and eventually retreated landward as the fluviodeltaic topset and foreset elongated. However, their timings of retreat were different, resulting in a counterintuitive case for a quicker retreat of GST while the shoreline still continued to advance. A series of scaled flume experiments with a sand and crushed walnut sediment mixture captured the same behaviours of these two moving boundaries. We found that GST experienced higher relative sea-level rise (RSLR) rates than the shoreline. This additional RSLR rate scales with the downstream river slope and the shoreline progradation rate to cause earlier GST retreat in comparison to the shoreline. The fundamental understanding from this study of migration of both the GST and shoreline in fluviodeltaic systems will aid in accurately assessing the trajectories of GST in sedimentary strata as a proxy for environmental change.

INTRODUCTION

One of the primary physical bases for identifying external forcing acting on basin evolution is grain-size variation in the sedimentary record. In modern river systems, grain size generally shows downstream fining due to selective sorting (e.g. Paola et al., 1992; Hoey & Ferguson, 1994; Cui et al., 1996; Wright & Parker, 2005; Fedele & Paola, 2007) and/or abrasion (e.g. Parker, 1991b; Le Bouteiller et al., 2011). The downstream fining is often locally abrupt and develops a narrow grain-size transition, e.g. the grain size suddenly changes from ~10 mm to ~1 mm over the ~40-km river distance along the River Rhine (Frings, 2011). The subsurface record, which is a time integration of the surface dynamics, reflects the spatial grain-size changes in the paleocurrent direction and also exhibits the temporal changes in grain-size boundary locations. Trains of lithofacies units are stacked vertically (in time) as they migrate downbasin (in space) such that the final strata show fining- or coarsening-upward sequences depending on the direction of the facies migration. As Walther’s law (Middleton, 1973) states, in the absence of depositional hiatuses and erosion, facies changes and associated grain-size changes in vertical succession can be translated into a lateral facies trend. Cycles of coarsening- and/or fining-upward successions can be indicative of forward and backward migration of the lithofacies train and thus indicative of advancement and retreat of the grain-size transitions.

Regression and transgression of the fluviodeltaic shoreline robustly indicates variations in basinal forcings (e.g. climatic variation, sea-level change and basin subsidence) (Helland-Hansen & Martinsen, 1996; Kim et al., 2006). However, paleo-shoreline records are often incomplete. Moreover, migrations of the lithofacies (grain-size) transitions have been treated as alternative stratigraphic indicators for changes in basinal controls, and many have interpreted it in a very similar fashion to the shoreline. For example, progradation of a fluviodeltaic system often corresponds to an increase in sediment supply into the basin due to active tectonics in the upland source area and/or a decrease in the relative sea-level rise rate, thus...
developing upward coarsening successions farther down-basin. One can hypothesize that the migrations of the internal grain-size transitions in this case are similar to the shoreline migration, and grain-size transitions should move in the same direction as the shoreline migrates, as is common in the conceptual models for a progradational parasequence set (e.g. Van Wagoner et al., 1988, 1990). In sequence stratigraphy, maximum flooding and maximum regressive surfaces in shallow siliciclastic basins are in general mapped at the top of fining-upward and coarsening-upward trends, respectively. As in the careful review of Catuneanu et al. (2009), identification of these surfaces based on grain size is simple, but may involve a significant uncertainty. The current study reproduced a grain-size transition that, under certain conditions, moves in a reverse direction against that of the shoreline and found the development of fining-upward sequences during shoreline progradation.

We present a fluviodeltaic-evolution model that rigorously treats both the shoreline and the grain-size transition (henceforth GST) as moving boundaries. This study uses a combination of numerical modelling and laboratory experiments that tests the responses of shoreline and GSTs under a range of relative sea-level rise (RSLR) rates. By capturing dynamic responses of these boundaries, we can provide quantitative understanding of the stratigraphic signatures of the moving boundary (delta shoreline) at the downstream end of a fluviodeltaic system and the upstream deltaic moving boundaries (grain-size transitions). The limited spatial distribution and temporal resolution of field data means it cannot provide information over the full extent of a depositional system or basin. However, these carefully scaled experiments can help to create a more complete picture. The results of this study should enhance the basis for correctly coupling the interpretation of local stratigraphic sections with basin-wide depositional history.

**MATHEMATICAL MODEL**

This new model couples the dynamics of the shoreline (i.e. Parker et al., 1998, 2008; Kim et al., 2009a,b) at the downstream end of the subaerial section of a delta and the grain-size transition at the middle of the subaerial section of a delta, by treating both points as moving boundaries (Parker & Cui, 1998; Parker, 2004) that respond to RSLR. Here we present formulas for the hybrid shoreline-GST model only briefly. The formulas are based on ones previously developed individually for shoreline and gravel–sand transition. A more detailed description for modelling the shoreline and the gravel–sand transition individually is available in the E-Book Chapters of Parker (2004).

Consider a fluviodeltaic system (Fig. 1) advancing into a standing body of water (e.g. lake or ocean). This model only employs a single grain-size transition point for simplification. Therefore, the fluviodeltaic surface is subdivided into three separate reaches in the downstream (x) direction: Gravel reach, sand reach, and delta front slope (or foreset). There are three moving boundaries in the model: the gravel–sand transition (x = e), the shoreline (x = s), and the delta toe (x = u). We assume no sediment transports beyond the delta toe – which means no bottom-set deposits exist – to focus on investigating the relationship between the gravel–sand transition and shoreline.

The Exner equations for sediment mass conservation on the gravel (indicated by the subscript g) and sand (indicated by the subscript s) reaches take the following combined form:

\[ \frac{\partial \eta_g}{\partial t} + \sigma = - \frac{I_t (1 + \Lambda_{s,g,ms}) \Omega_{g,s,e} \delta \Omega_{g,s,c}}{(1 - \lambda_{ps,pg}) \Phi_{g,s}} \frac{\partial \eta_s}{\partial x}, \]

where \( t \) denotes time; \( \eta_g \) and \( \eta_s \) represent sediment surface elevation on gravel and sand reaches respectively; \( \sigma \) is the subsidence rate; \( I_t \) represents flood intermittency; \( \Omega_{g,s} \) and \( \Omega_{g,s,e} \) are channel sinuosity on gravel and sand reaches; \( \lambda_{ps,pg} \) and \( \lambda_{ps,pg} \) represent bed porosity of gravel and sand reaches; \( \Phi_{g,s} \) and \( \Phi_{g,s,e} \) are ratios of channel width \( B_t \) to depositional width \( B_s \) (total basin or floodplain width) for the gravel and sand reaches respectively; \( x \) is a spatial coordinate in the downstream direction; \( \delta \) and \( \delta_s \) are the total volume for gravel and sand loads per unit width respectively; and \( \Lambda_{s,g} \) and \( \Lambda_{s,g} \) denote volume fraction of sand deposited per unit sand and volume fraction of mud deposited per unit sand in the channel-floodplain complex respectively.

Bed elevation continuity at the gravel–sand transition (\( x = e \)) can be expressed as \( \eta_{g,s} (e(t), t) = \eta_{g,s} (t, t) \). The

![Fig. 1. Schematic of a delta with three moving boundaries, i.e. gravel–sand transition (\( e = GST \)), shoreline (s) and delta toe (u), along with other parameters used in the model.](image-url)
time derivative of this continuity relation yields the following form for the migration speed of the gravel–sand transition:

\[ \dot{e} = \frac{\partial n_1}{\partial t} + \frac{\partial n_1}{\partial x} \left[ S_{\text{sl}} - S_{\text{sk}} \right]. \]  

(2)

where the dot on \( e \) denotes the first derivative with respect to time, and \( S_{\text{sk}} \) is the gravel-bed slope at the transition and \( S_{\text{sl}} \) is the sand-bed slope at the transition. Equation (2) indicates that the GST migration is controlled by the competitions of relative depositional rates and surface slopes between the gravel and sand reaches. The sediment conservation equation (1) can be integrated across the delta front to yield a shock condition such that no sand escapes the delta toe beyond \( x = u \). Bed elevation, \( \eta_1 \), across the foreset \((u < x < s)\) can be written as \( \eta_0 = \eta_1[x(t), t] - S_{\text{sk}}(x - s(t)) \). Substituting a first-time derivative of this foreset profile into the shock condition yields a solution for the shoreline migration rate as

\[ \dot{s} = \frac{1}{(S_{\text{f}} - S_{\text{sk}})} \left( \frac{I(t)}{(1 - \lambda_{\text{ms}})(u - s)} + \frac{\partial n_1}{\partial t} + \frac{\partial n_2}{\partial t} \right). \]  

(3)

Here the shoreline migration rate is mainly determined by the sediment flux at the shoreline and relative sea-level rise. The foreset terminates downdip against the basement surface, and thus the bottom elevation of the foreset matches continuously with the basement elevation \( \eta_0 \) at \( x = u \) as \( \eta_0[u(t), t] = \eta_1[x(t), t] - S_{\text{sk}}[u(t) - s(t)] \). The study assumes a nonerodible basement of uniform slope without tectonic subsidence. Taking the derivative with respect to time of the equation yields a moving boundary solution for the delta toe as

\[ \dot{u} = \frac{1}{(S_{\text{f}} - S_{\text{sk}})} \left( \frac{\partial n_1}{\partial t} + (S_{\text{f}} - S_{\text{sk}}) \dot{s} \right). \]  

(4)

A backwater formulation is used to compute the flow. The friction coefficients on the gravel and sand reaches, correspondingly \( C_{\text{gk}} \) and \( C_{\text{as}} \), are assumed to be specified constants. The backwater formulations for the gravel reach and sand reach are combined in following formula:

\[ \frac{\partial H_{\text{g,s}}}{\partial x} = \frac{S_{\text{g,s}} - C_{\text{g,k}} F_{\text{g,s}}^2}{1 - F_{\text{g,s}}^2}, \]  

(5)

where \( F_{\text{g,s}} \) and \( F_{\text{g,k}} \) denote the Froude number for the gravel and sand reaches, respectively, and \( H \) is the flow depth. The flow depth at the shoreline is defined as \( H_{\text{sk}} = Z_o + \zeta - \eta_{\text{sk}} \), where \( Z_o \) denotes the initial sea level and which may change in time at some constant rate, \( \zeta \). In addition, a continuity condition must be satisfied at the gravel–sand transition as \( H_{\text{g}} = H_{\text{sk}} \).

In the present implementation, gravel transport on the gravel reach is calculated using the Parker (1979) approximation of the Einstein (1950) relation,

\[ q_{\text{g}} = \sqrt{R g D_{\text{g}} D_{\text{g}} 11.2 (\frac{\tau_{\text{g}}^{\text{c}}}{\epsilon})^{1.5} \left(1 - \frac{0.03}{\tau_{\text{g}}^{\text{c}}} \right)^{4.5}}. \]  

(6)

Sand transport on the sand reach is calculated using the Engelund & Hansen (1972) formulation,

\[ q_{\text{s}} = \sqrt{R g D_{\text{s}} D_{\text{s}} 0.05 \left(\tau_{\text{g}}^{\text{c}}\right)^{2.5}}. \]  

(7)

**MODELLING RESULTS**

The model was first used to investigate the behaviours of the gravel–sand transition and the shoreline in a large-scale fluviodeltaic system. Three scenarios with constant RSLRs = 2, 6, and 10 mm year\(^{-1}\) were tested. Modelling runs are referred to herein as “M1”, “M2”, and “M3”, respectively. Initial sediment surface elevation at the shoreline was set at 0 m, initial channel depth at the shoreline was set at 5 m as well as the initial sea level, and initial channel width was 100 m. All other parameters, e.g. sediment discharges for gravel and sand = 0.1 and 0.2 m\(^3\) s\(^{-1}\), bankfull water discharge = 1000 m\(^3\) s\(^{-1}\), and grain sizes for gravel and sand = 25 and 0.25 mm, respectively, were kept constant across these three modelling runs (Table 1).

Figure 2 shows the cross-sectional view of a delta developed over 1000 years in response to RSLR of 2 mm year\(^{-1}\) (M1: Fig. 2a), 6 mm year\(^{-1}\) (M2: Fig. 2b), and 10 mm year\(^{-1}\) (M3: Fig. 2c). Each cross-section includes strata composed of 20 timelines created every 50 years. All the runs start with a pre-existing delta with the gravel–sand transition at \( x = 15 \) km and the shoreline at \( x = 30 \) km, which divides the fluviodeltaic surface into an upstream gravel reach, a downstream sand reach and a foreset each with a distinct topographic slope. Gravel and sand reaches are initially assigned slopes of 0.001 and 0.0001, respectively, but self-organized over time by the sediment transport processes in the model. The foreset slope is assigned as \( S_{\text{f}} = 0.2 \) and remains constant over time. The shoreline is located at the break in slope between the sand reach and foreset slope. The gravel reach is steeper than the sand reach with a clear break in slope between the two reaches. The delta front advances over a flat basement at \( -10 \) m. Overall, the shoreline advances seaward. The degree of upward convexity in the shoreline trajectory corresponds to different RSLRs applied to the run. For example, high convexity in the shoreline trajectory represents the stratigraphic signature of high RSLR (Helland-Hansen & Martinsen, 1996). Moreover, the migration pattern in the GST generally mimics that of shoreline. The gravel–sand transitions in M1 and M2 initially advance in the downbasin direction with a higher initial rate that decreases over time. The early stage of gravel–sand transition in M3 is similar to M1 and M2; however, the transition eventually starts to retreat landward while the shoreline keeps proggrading.
As a consequence of this retreat, vertical sections found near the gravel–sand transition would exhibit a fining-upward sequence in this overall prograding system.

Nondimensionalizing the control parameters facilitates the comparison of systems with different fluviodeltaic and basin forcing parameters. A fluviodeltaic system that develops on a linearly sloping basement (i.e. shelf or ramp) has no physiographic length scale (Swenson & Muto, 2007). A length scale can be characterized from the sediment supply and the RSLR rate:

\[ L = \frac{q_T}{|f|^{2/a}} \]

(2)

where \( q_T \) is the total sediment discharged to the basin per unit width. One natural choice for the time variable is

\[ T = \frac{q_T}{|f|^{2/a}} \]

(3)

where \( a \) denotes a geometric constant describing

\[ \frac{1}{S_t} - \frac{1}{S_b} \]

Table 1. Modelling parameters used in M1-M3

<table>
<thead>
<tr>
<th>Parameter ( Q_{bf} )</th>
<th>Input value(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{fg} )</td>
<td>0.05</td>
<td>Flood intermittency, gravel-bed reach</td>
</tr>
<tr>
<td>( I_{fs} )</td>
<td>0.05</td>
<td>Flood intermittency, sand-bed reach</td>
</tr>
<tr>
<td>( Q_g )</td>
<td>0.1</td>
<td>Feed rate of gravel (m³ s⁻¹)</td>
</tr>
<tr>
<td>( Q_s )</td>
<td>0.2</td>
<td>Feed rate of sand (m³ s⁻¹)</td>
</tr>
<tr>
<td>( L_{bg} )</td>
<td>100</td>
<td>Initial gravel-bed channel width (m)</td>
</tr>
<tr>
<td>( L_{bs} )</td>
<td>100</td>
<td>Initial sand-bed channel width (m)</td>
</tr>
<tr>
<td>( L_{bg} )</td>
<td>5000</td>
<td>Depositional or floodplain width, gravel-bed reach (m)</td>
</tr>
<tr>
<td>( L_{bs} )</td>
<td>5000</td>
<td>Depositional or floodplain width, sand-bed reach (m)</td>
</tr>
<tr>
<td>( \Omega_g )</td>
<td>2</td>
<td>Sinuosity of gravel-bed reach</td>
</tr>
<tr>
<td>( \Omega_s )</td>
<td>2</td>
<td>Sinuosity of sand-bed reach</td>
</tr>
<tr>
<td>( A_{fg} )</td>
<td>0.5</td>
<td>Fraction of sand deposited per unit gravel in depositional</td>
</tr>
<tr>
<td>( A_{fs} )</td>
<td>0.5</td>
<td>Fraction of mud deposited per unit sand in depositional</td>
</tr>
<tr>
<td>( D_g )</td>
<td>25</td>
<td>Grain size of gravel (mm)</td>
</tr>
<tr>
<td>( D_s )</td>
<td>0.25</td>
<td>Grain size of sand (mm)</td>
</tr>
<tr>
<td>( C_{zg} )</td>
<td>15</td>
<td>Dimensionless Chezy resistance coefficient gravel-bed reach</td>
</tr>
<tr>
<td>( C_{zs} )</td>
<td>25</td>
<td>Dimensionless Chezy resistance coefficient sand-bed reach</td>
</tr>
<tr>
<td>( L )</td>
<td>30 000</td>
<td>Initial reach length (m)</td>
</tr>
<tr>
<td>( e/l )</td>
<td>15 000</td>
<td>Initial position of gravel–sand transition (m)</td>
</tr>
<tr>
<td>( S_{lg} )</td>
<td>0.0011</td>
<td>Initial slope of gravel-bed reach</td>
</tr>
<tr>
<td>( S_{ls} )</td>
<td>0.00011</td>
<td>Initial slope of sand-bed reach</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0</td>
<td>Subsidence rate (mm year⁻¹)</td>
</tr>
<tr>
<td>( Z_o )</td>
<td>5</td>
<td>Initial base level (m)</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>2–10</td>
<td>Rate of base-level rise (mm year⁻¹)</td>
</tr>
<tr>
<td>( dT )</td>
<td>0.2</td>
<td>Time step (year)</td>
</tr>
<tr>
<td>( \eta_{bf} )</td>
<td>0</td>
<td>Initial elevation of top of the foreset (m)</td>
</tr>
<tr>
<td>( \eta_{bf} )</td>
<td>−10</td>
<td>Initial elevation of bottom of the foreset (m)</td>
</tr>
<tr>
<td>( S_f )</td>
<td>0.2</td>
<td>Slope of foreset surface</td>
</tr>
<tr>
<td>( S_b )</td>
<td>0</td>
<td>Slope of subaqueous basement</td>
</tr>
<tr>
<td>( \lambda_g )</td>
<td>0.4</td>
<td>Bed porosity, gravel-bed reach</td>
</tr>
<tr>
<td>( \lambda_s )</td>
<td>0.4</td>
<td>Bed porosity, sand-bed reach</td>
</tr>
<tr>
<td>( R )</td>
<td>1.65</td>
<td>Submerged specific gravity of sediment</td>
</tr>
</tbody>
</table>

Swenson, 2005), where \( q_T \) is the total sediment discharged to the basin per unit width. One natural choice for the time variable is \( T = \frac{q_T}{|f|^{2/a}} \) (Swenson, 2005), where \( a \) denotes a geometric constant describing \( (1/S_t - \frac{1}{S_b}) \).
Grainsize transition with sea-level rise

Because the model sets \( S_f \) as a constant, we use the assigned \( S_f \) for this analysis but take the average slope between the gravel and sand reaches at the final stage for \( S_t \). In M3 e.g. the characteristic length and time scales for the basin are \( L = \sim 47 \) km and \( T = \sim 1400 \) years respectively. Figure 3 shows the results of migration trajectories of the two boundaries in dimensionless units. M1 and M2 (Fig. 3a,b) represent an immature stage of basin growth, whereas M3 (Fig. 3c) closely reaches the full-growth stage, where shoreline progradation significantly slows and aggradation closely balances with RSLR.

**EXPERIMENTAL TEST OF THEORY: DESIGN**

A series of six experiments (hereafter R1-6; Table 2) was conducted at the University of Texas using a 3.5-cm-wide flume (Fig. 4: 0.88 m long × 3.5 cm wide × 0.5 m deep). The experiments used a sediment mixture composed of well-sorted, crushed walnut shell (brown) sediment and quartz (white) sand at a two to one ratio in volume. Crushed walnut shell sediment has a median grain size of 0.15 mm and density of 1300 kg m\(^{-3}\). Quartz sand has a similar grain size of 0.17 mm, but higher density of 2650 kg m\(^{-3}\). The sediment mixture and water were mixed in a funnel outside of the flume and introduced into the flume through a steep open channel that terminated at the far, upper corner of the basin. Each experiment started with an initial base level of 5 cm. Sediment discharge and water discharge were kept constant for all the runs at \( Q_s = 3.34 \) g s\(^{-1}\) and \( Q_w = 11.39 \) mL s\(^{-1}\). For the initial 3 min (=180 s) of each run, no base-level change was applied. Here the transitional boundary between the quartz sand (a proxy for gravel) reach and walnut sediment (a proxy for sand) reach works as a proxy for the gravel–sand transition. This grain-size transition (GST) gradually migrated downstream by a distance equivalent to the half of the distance that the shoreline advanced during the initial 3-min interval (Fig. 5a). Runs 1-6 had RSLRs of 0, 0.007, 0.013, 0.052, 0.116, and 0.325 mm s\(^{-1}\), respectively applied at run time = 3 min until the end of each run. These parameters were chosen to yield experimental results with a relatively wide range in \( L \) and \( T \) scales for the experimental limitations (e.g. size of the flume, discharge ranges, maximum and minimum base-level change rates, etc.).

Time-lapse images in cross-sectional view were taken every 15 s during the experiments. The images were corrected for lens distortion and perspective before mapping the shoreline and GST positions (Tal et al., 2012). At the end of each run, surface topography was measured to provide a means to verify the analysed data from the corrected images.
Table 2. Experimental parameters used in runs R1-R6

<table>
<thead>
<tr>
<th>Index</th>
<th>Total run time (s)</th>
<th>Sea-level rise rate (mm s(^{-1}))</th>
<th>Initial sea level (mm)</th>
<th>(Q_s) (g s(^{-1}))</th>
<th>(Q_w) (cm(^3) s(^{-1}))</th>
<th>(Q_w) (cm(^3) s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>480</td>
<td>0</td>
<td>50</td>
<td>3.34</td>
<td>1.91</td>
<td>11.39</td>
</tr>
<tr>
<td>R2</td>
<td>525</td>
<td>0.072</td>
<td>50</td>
<td>3.34</td>
<td>1.91</td>
<td>11.39</td>
</tr>
<tr>
<td>R3</td>
<td>630</td>
<td>0.013</td>
<td>50</td>
<td>3.34</td>
<td>1.91</td>
<td>11.39</td>
</tr>
<tr>
<td>R4</td>
<td>660</td>
<td>0.052</td>
<td>50</td>
<td>3.34</td>
<td>1.91</td>
<td>11.39</td>
</tr>
<tr>
<td>R5</td>
<td>915</td>
<td>0.116</td>
<td>50</td>
<td>3.34</td>
<td>1.91</td>
<td>11.39</td>
</tr>
<tr>
<td>R6</td>
<td>540</td>
<td>0.325</td>
<td>50</td>
<td>3.34</td>
<td>1.91</td>
<td>11.39</td>
</tr>
</tbody>
</table>

EXPERIMENTAL TEST OF THEORY: EXPERIMENTAL RESULTS AND MODEL PREDICTIONS

For the first minute of the experiment, a triangular, submerged foreset grew without a subaerial topset. Once the subaerial delta topset developed above the base level, the topset deposit began accumulating and included mostly sand, whereas the submerged foreset was mainly composed of walnut sediment. The GST overlaid the shoreline in this initial stage. The GST separated from the shoreline at about run time = 2 min. The shoreline prograded faster than the GST, which resulted in equal shoreline and GST delta was built with a white sand and brown walnut sediment mixture supplied from the right-hand side of the flume.

The flux relation takes the following form:

\[
\frac{Q_s}{Q_w} = \alpha S^n, \tag{9}
\]

where \(\alpha\) is a dimensionless coefficient and \(n\) is an exponent. These values are determined for both the sand and walnut sediment reaches, using separate flume experimental runs conducted with different sediment and water discharges (Fig. 7). Regression of these data points gives \(\alpha = 111.31\) and \(n = 3.62\) for sand reach and \(\alpha = 359.97\) and \(n = 2.46\) for walnut sediment reach. The calculated slope exponent falls within the previously reported 1.5–4.5 range (e.g. Swenson & Muto, 2007; Parker \textit{et al.}, 2008; Kim \textit{et al.}, 2009a) for sediment of similar grain size and flows with similar sediment and water discharges.
Numerical model predictions for R1-6 are also shown in Fig. 6. The modelling runs captured changes in GST and shoreline positions from run time $t = 180$ s to the end of each experiment. The initial locations for the GST and shoreline measured at run time $t = 180$ s in each experiment were used for modelling the initial condition. Simplifying the calculations with the slope-driven sediment transport relations (9) omits the backwater formulation from the model. In the modelling of the experiments, the morphodynamic activity was assumed to be continuous, i.e. $I = 1$ in (1), the bed porosity $\lambda_p$ was 0.5, no subsidence occurred (i.e. $\sigma = 0$), no floodplain existed (i.e. $\Phi = 1$), no channel sinuosity existed (i.e. $\Omega = 1$), and walnut sediment and quartz sand were perfectly sorted (i.e. $\Lambda_{sg}$ and $\Lambda_{ms} = 0$; upstream of the GST contains only walnut sediment deposits and downstream of the GST contains only sand).

Experimental runs (R1-6) provide a test of the model predictions for GST and shoreline migration in response to steady RSLR. The predictions from the model track the experimental data well for all runs. While the details of the modelling results in Fig. 6 are specific to the present experiments, it is important to note that the overall conclusion is independent from the empirical sediment transport relation used in this model. The modelling results thus capture the first-order response of GST and shoreline migration over a range of RSLR rates. The results also show GST and shoreline response to RSLR at different rates. Furthermore, the GST and shoreline can move in opposite directions in response to uniformly applied RSLR. For field application, (9) can be again switched back to (6) and (7), or to another set of sediment transport relations that are appropriate to the field conditions for the different grain-size reaches in question.

**DISCUSSION**

Reversal in the GST to shoreline migration

One might expect that the GST and shoreline (and/or clinoform rollover) could advance or retreat at different rates in response to sea-level changes. However, except for inherent variability at small temporal and spatial scales, previous studies have not predicted that these boundaries move in opposite directions and thus produce fining-upward and coarsening-upward trends within the same stratigraphic interval without any local changes in tectonics, base level and sediment supply. Both basin-scale numerical modelling and scaled flume experiments indicate that GSTs can migrate at different rates and also...
migrate in the opposite direction to the shoreline (/clinoform rollover). RSLR, sediment supply and water discharge were constant in each modelling and experimental run, and the 1:2 ratio in the sand to walnut sediment mixture did not change either. What actually caused the reversal of the GST? More specifically, why did the GSTs in M3 and R5 start to retreat landward before the shoreline?

To answer this question, it would be worth briefly reviewing the autoretreat theory developed for shoreline migration in response to RSLR. Consider a fluviodeltaic system with sloped topset and foreset. As a delta grows with RSL rising at a constant rate, the shoreline progradation rate decreases with time and eventually the shoreline ceases to advance basinward and instead retreats landward (Muto & Steel, 1992; Swenson et al., 2000; Muto, 2001;
Kim et al., 2006; Kim & Muto, 2007). This occurs without any changes in RSLR rate. The topset and foreset increase in length as the delta progrades and aggrades. To maintain the same rate in shoreline regression, the growing delta requires more sediment. With a constant sediment supply to the system, it is inevitable the delta experiences a decrease in shoreline progradation rates and eventual retreat. Decrease in shoreline progradation rate is intensified with a RSLR rate increase or diminished with a sediment supply increase. As indicated in Figs 2 and 6, higher RSLR rates induce higher convexity in the shoreline trajectory, representing a rapid reduction in progradation rate. The GST mimics the same pattern of migration as the shoreline. However, the retreat of the GST occurs sooner than that of the shoreline because the GST experiences greater base-level rise compared to the sea-level rise at the shoreline. This differential sea-level rise for the GST is a by-product of shoreline progradation in a delta with a sloped topset. Figure 8 shows the difference in base-level rise at the GST and shoreline progradation in a delta with a sloped topset. Figure 8 shows the difference in base-level rise at the GST position compared to sea-level rise at the shoreline and shows an increase in RSLR scales with the topset slope and shoreline progradation rate. Strong progradation during the early stage of delta growth increases RSLR at the GST and therefore shortens the time necessary for the GST to initiate retreat compared to shoreline. The initial fast regression of the shoreline in addition to the rapid RSLR in M3 and R5 (Figs 3c and 6e) caused the GST to reverse its course earlier and migrate in a direction counter to the migration of the shoreline. This phenomenon best explains the stratigraphic intervals in parasequence set and parasequence scales. The process presented in the study, however, occurs equally to any time and spatial scales as it is demonstrated in the small-scale experiments and also field-scale modelling at approximately $r^* = 0.1 - 1$ and $x^* = 0.1 - 1$.

**Other possible model generalizations**

A fair number of simplifying assumptions were used in the model including two mean grain sizes for gravel and sand, uniform floodplain width, constant channel sinuosity, constant channel forming bankfull discharge with an intermittency and linear relative sea-level rise (Table 1). In our model, channel width and depth are self-organized in response to the imposed boundary conditions, yet the model can form only one channel and the predictions for delta morphology represent only cross-basin averages. In order to capture other natural processes, the model requires generalization. For example, the sediment transport relation (Parker, 1979) used for the gravel reach could be replaced by a relation that explicitly models grain-size variation and captures downstream fining (e.g. Parker, 1991a,b). The moving boundary method (Swenson et al., 2000; Voller et al., 2006) used here to model the shoreline and GST migration has proven to capture the dynamic changes in the shoreline and GST locations.
in response to relative sea-level rise. For the foreset, we used a simple linear slope most appropriate for a steep gilbert-type delta front built by grain avalanching. The model also could include a bottomset at the base of the foreset and use a sediment transport relation for plunging turbidity currents (Kostic & Parker, 2003) and introduce yet another level of complexity into the model. Additionally, deltas can grow in different planform patterns, e.g. radially with an opening angle. In this radial growth, a delta can be modelled as an axially symmetric, conical-

Fig. 9. Time series of GST and shoreline positions in (a) R2, (b) R3, (c) R4, (d) R5 and (e) R6 in dimensionless units and (f) a new R4 run with a new parameter set that produces a similar evolution pattern to M3.
Grainsize transition with sea-level rise

shaped deposit (Kim et al., 2009a,b). While further generalizations could capture more details in delta evolution and would be interesting, they cost more computational power and introduce more complexity into the analysis. In fact, the general conclusion of the GST and shoreline migration in response to RSLR does not change with these generalizations of the model.

CONCLUSIONS

Traditionally, it has been widely accepted that shoreline and associated lithofacies boundaries (which may be approximated by grain-size transitions) share a similar migration pattern in response to relative sea-level changes. Here we present a theoretical model and flume experiments that demonstrate cases where these boundaries do not migrate in the same direction despite constant sediment and water supply.

In both the numerical model and experiments, the GST boundary can initiate its landward retreat in response to RSLR before the shoreline does. The earlier timing of the GST retreat arises because the GST experiences an exaggerated RSLR relative to the shoreline as a by-product of progradation in a delta with a sloped topset. Therefore, the same stratigraphic interval can show both fining-upward and coarsening-upward trends during an overall progradation. Additionally, the pattern that the GST follows in order to converge into a stable position depends on the boundary conditions, i.e. the initial GST and shoreline positions. If the initial delta is small before sea-level rise begins, rapid progradation of the GST is unlikely.

ACKNOWLEDGEMENTS

Data for this paper are available through the Sediment Experimentalists Network (SEN) Knowledge Base. This work was supported by NSF grant EAR 1148005 to W. Kim. We thank Sebastien Castelltort, Gary Hampson and an anonymous reviewer for their constructive comments on the paper.

CONFLICT OF INTEREST

No conflict of interest declared.

REFERENCES


Parker, G. (1991a) Downstream variation of grain size in gravel rivers: abrasion versus selective sorting, fluvial hydraulics of mountain regions. In: Lecture Notes in Earth Sciences (Ed. by

Manuscript received 8 January 2016; In revised form 21 May 2016; Manuscript accepted 30 June 2016.