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Spatial peculiarity of graded alluvial rivers in deltaic settings: 2D tank experiments

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ABSTRACT

Stage of a river that conveys sediment without net deposition and net erosion is referred to as ‘graded’ with respect to vertical aggradation of the river segment. Three experimental series, designed in terms of the autostratigraphic view of alluvial grade, were conducted to bring a clarification of diagnostic spatial behavior of graded alluvial rivers in deltaic settings: R series as a reference under a moving boundary condition with stationary base level, F series under a fixed boundary condition with stationary base level to produce ‘forced grade,’ and M series under a moving boundary condition with constant base level fall to produce ‘autogenic grade.’ The results of the three experimental series, combined with geometrical modeling of the effects of basin water depth and other experimental data, suggest a whole new understanding that (1) in a graded alluvial-deltaic system, lateral shifting and avulsing of active distributary channels are suppressed regardless of whether or not the downstream boundary of the deltaic system is fixed, (2) in a delta having a downstream boundary of the deltaic system is fixed, the graded streams are stabilized within a valley incised in the axial part of the delta plain, whereby the alluvial plain outside the valley is abandoned and terraced, (3) under a moving boundary condition, the graded river simply extends basinward as a linearly elongated channel-lobe system, without cutting a valley, and (4) a modern forced-graded alluvial river is most likely found in a valley incised to a fan delta located in front of very deep water, and the stratigraphic sign of fossil autogenic-graded rivers will be found in deltaic successions which accumulated in outer to marginal areas of deltaic continental shelves during sea level falls. This 2D autostratigraphic
view of alluvial grade suggests thorough reconsideration of the conventional understanding of intrinsic stratigraphic responses of alluvial-deltaic systems to base level forcing.

**Keywords** alluvial river, autostratigraphy, base level, delta, experiment, grade

**INTRODUCTION**

Stage of a river that conveys sediment without net deposition and net erosion is referred to as ‘graded’ with respect to vertical aggradation of the river segment. Grade is thus the incarnation of a dynamic equilibrium state of the river as to sediment balance. The concept of grade, which was originally advocated by Gilbert (1877), is typically presented as the long-term, equilibrium state of a river system subject to steady external forcing by stationary base level (Davis, 1902; Green, 1936; Kesseli, 1941; Leopold & Bull, 1979; Posamentier & Vail, 1988; Thorne & Swift, 1991; Johnson & Beaumont, 1995; Holbrook et al., 2006), with special emphasis on the rivers’ adjustments in slope (Mackin, 1948; Schumm, 1977). A correct understanding of grade is fundamental to the morphodynamics, sedimentology and geology of river deltas, because grade represents the critical condition that discriminates between aggradational and degradational regimes in a river system, and also because grade is the key to the exploration of intrinsic river response to base level forcing. Conventional notions in geology suggest that stratigraphic responses of an alluvial river to base level changes are controlled by the graded profile of the river (e.g., Posamentier et al., 1988). Sequence stratigraphy, together with ancestral concepts advocated in the 1960s and earlier, has been developed on the assumption that (1) rivers basically aggrade in response to base-level rise and degrade in response to base-level fall, and (2) grade is the final, stable state of a river system that is attained through equilibrium response to stationary base level (Posamentier & Vail, 1988; Thorne & Swift, 1991a; Holbrook et al., 2006). For this latter concept, equilibrium response refers to a type of response by which steady stratigraphic configuration arises from steady dynamic external forcing or static external forcing (Muto & Steel, 2014).

Recent studies of alluvial-deltaic evolution as a moving boundary problem have brought a drastic revision of the grade concept, including notions that (1) the feeder alluvial river can become graded only during base-level fall (Nummedal et al., 1993; Leeder & Stuart, 1996), (2) time patterns of base-level fall to allow the river to become graded depend on basin configuration (Muto et al., 2014), and (3) alluvial rivers can be graded both allogenically and autogenically (Muto & Swenson, 2006). This set of novel notions is referred to as the
‘autostratigraphic view of grade’ (Muto et al., 2007; see below) and has been derived largely from 1D geometrical modeling and corroborative 1D narrow flume experiments. However, a few critical questions remain unsolved, such as how a graded river behaves in a 2D spatial setting, what planometric geometry and channel patterns could be intrinsic to a graded river, and what stratigraphic and/or geomorphic evidence is left by a graded river in geological records. Although a graded river, by definition, leaves no deposits and has no erosional features, there is no established rationale for denying the possibility that indirect signs of its spatial and temporal presence can be detected from the landscape or the strata.

The objective of the present study is to extend what has been learned in 1D studies to 2D modeling of alluvial grade through tank experiments. Any depositional system actually has 3D spatial extent, but our primary focus here is on the macroscopic behavior of graded streams in the planar surface. For this reason, the term ‘2D’ is used in the present study. As shown below, the results of 2D experiments, combined with simple geometrical modeling and existing notions obtained from a previous series of experiments, make it convinced that 2D graded alluvial-deltaic systems can behave in distinctive spatial patterns that will be identifiable in both modern and ancient river deltas.

**AUTOSTRATIGRAPHIC VIEW OF ALLUVIAL GRADE**
Autostratigraphy is the stratigraphy generated by large-scale autogenic processes, based on the full recognition of (1) the nonequilibrium behavior of depositional systems in response to steady external forcing and (2) the general lack of equilibrium configuration of depositional systems (Muto et al., 2007). Nonequilibrium response is a type of response by which steady dynamic forcing results in unsteady stratigraphic configuration of the depositional system, or unsteady dynamic forcing is responsible with steady stratigraphic configuration (Muto & Steel, 2014). It is a nonequilibrium response rather than an equilibrium response that generally holds under steady external forcing. Though an equilibrium response is physically possible as noted before (Muto & Swenson, 2006) and is also illustrated in the present study, ignoring the nonequilibrium response can lead to a serious misinterpretation of stratigraphic records.

1D experiments performed over the past decade have clarified that in a moving boundary system, alluvial grade can be attained in the case of either nonequilibrium or equilibrium responses, but it requires base-level fall (Muto & Swenson, 2006). Thus, the critical condition for discriminating between aggradational and degradational regimes of alluvial systems is not the stationary base level but a continuous fall in base level. Alluvial rivers can, in principle, continue to aggrade throughout the duration of base-level fall unless the base-level falls below
the grade curve (Petter & Muto, 2008).

A rigorous stratigraphic criterion to judge whether or not an alluvial river in a deltaic setting is graded is the lack of an alluvial topset deposit despite no significant erosional features in the delta profile. An alternative, easier method is to monitor the behavior of the alluvial-basement transition (ABT), i.e. the upstream end of the alluvial reach (Fig. 1). The alluvial river is aggrading if ABT is migrating upstream, and it is degrading if ABT is migrating downstream. Thus, a condition for the river to remain graded is for ABT to remain stationary. The sustained standstill of ABT is thus a diagnostically necessary condition for achieving alluvial grade. This easy method is adopted in the present experimental study where a stratigraphic cross section was not available.

Under the moving boundary condition, alluvial grade can arise in two different ways, depending on geomorphic conditions and time patterns of base-level fall for which alluvial slope $\alpha$ and basin floor slope $\phi$ are particularly influential (Figs. 1A, B). These two slope parameters affect the fate of sediment that has bypassed the alluvial reaches and accumulates basinward to the shoreline, which then feeds back to the feeder alluvial river (Petter & Muto, 2008).

Where $\alpha < \phi$, the delta progrades into water that progressively deepens basinward. Assume that sediment discharge $Q_s$ at ABT is always constant. For the alluvial river to retain a complete sediment-bypass system, base level is required to fall at a decelerating rate in a particular pattern (Muto & Swenson, 2005). Thus, the alluvial grade to be attained under this slope condition is an allogenic one that arises from nonequilibrium response to unsteady external forcing (‘allogenic grade’). If base level instead falls at a constant rate in this slope setting, the river aggrades at an early stage but later degrades by autogenic nonequilibrium response (Swenson & Muto, 2007).

Where basin slope is equal to alluvial slope ($\alpha = \phi$), water depth in front of the prograding delta does not change over time, provided that base level continues to fall at a constant rate. This slope condition is possible where a delta progrades above a drowned alluvial plain or an antecedent graded alluvial reach that was built during base-level fall or lowstand of the previous cycle. Under this topographic condition, alluvial grade is autogenically attained and sustained by an equilibrium response to the steady base-level fall (‘autogenic grade’; see Muto & Swenson, 2006). The rate of base-level fall simply functions to determine the set thickness of the prograding deposit, and it never affects whether or not grade is attained, as far as the rates of base-level fall and sediment supply are kept constant.

Where alluvial slope is greater than basin slope ($\alpha > \phi$), water depth in front of the delta decreases as it progrades with constant base-level fall, and the alluvial river eventually
detaches from the seaward-receding shoreline and thus becomes nondeltaic (Petter & Muto, 2008). In this slope setting along with constant base-level fall and constant sediment supply, the river can never attain grade but can sustain alluvial aggradation.

If the downstream end of the depositional system is fixed, alluvial grade can forcefully be attained with stationary base level (‘forced grade’: Fig. 1C), just as for cases of a nondeltaic alluvial river that is dammed up with a downstream weir (Postma et al., 2008; Cantelli & Muto, 2014) and for a deltaic alluvial river that is perched on a ‘shelf edge’ at which the feeder flows simply dump the entire supplied sediment into very deep water (Parker, 1977; Kim et al., 2013).

Thus, alluvial grade can arise by three different mechanisms: (1) forced grade attained through equilibrium response to stationary base level in a downstream-fixed setting, (2) allogenic grade attained through nonequilibrium response to decelerating base-level fall in a moving boundary setting where $\alpha < \phi$, and (3) autogenic grade attained through equilibrium response to constant base-level fall in a moving-boundary setting where $\alpha = \phi$. The present experimental study excludes allogenic grade because its 2D attainment is technically difficult through available experimental methods, though it is not impossible in principle.

**TANK EXPERIMENTS**

Tank experiments were conducted to explore the spatial behavior of a graded alluvial river under a basement condition that does not confine the delta’s progradation to the transverse direction. Three series of experiments were conducted: the ‘R series’ acts as a reference for the other two and was conducted in a moving boundary setting with stationary base level (Runs R1, R2, R3); the ‘F series’ examines the attainment of forced grade in a fixed boundary system with stationary base level (Runs F1, F2, F3, F4); and the ‘M series’ examines the attainment of autogenic grade in a moving boundary system with constant fall of base level (Runs M1, M2). All of the experiments were carried out in tank facilities at Nagasaki University. The sediment used in all runs was 0.2 mm uniform quartz sand (grain density = 2.63 g cm$^{-3}$, bulk density = 1.52 g cm$^{-3}$, porosity = 0.42). During each run in all of the series, upstream sediment discharge $Q_s$ and upstream water discharge $Q_w$ were kept constant. Runs R1–3, F1–4 and M1 were conducted in experimental tank ‘Margi 2’ (200 cm long × 100 cm wide × 70 cm deep), and Run M2 was conducted in ‘Margi 4’ (300 cm long × 600 cm wide × 55 cm deep). Experimental conditions are summarized in Table 1.

Plan view photos of the growing delta were taken every 30 seconds during each run. A primary measurement was to identify active shoreline segments on time-lapse images, i.e. the mouths of active distributary channels. Because distributary channels themselves are difficult
to identify in photographic images due to the clean water flow over the uniform sediment, our observation focused on detecting active channel mouths (i.e. active shoreline segments) that can be identified with ImageJ64 by comparing two successive images. The location of an active shoreline segment can be quantified through analysis of the azimuth angles $\theta$ ($0^\circ \leq \theta \leq 180^\circ$) at both ends of the segment measured from the initial source point, i.e. the intersection of an initial bedrock river with an initial shoreline, such that $\theta = 90^\circ$ for due basinward, $\theta = 0^\circ$ and $\theta = 180^\circ$ for the shoreline at either end of the two sides (Fig. 2). Pairs of measured values of $\theta$ were plotted against time $t$, and each segment was expressed as a horizontal bar having a width specified by the difference between values of $\theta$ (e.g., $\theta_2-\theta_1$, $\theta_4-\theta_3$, $\theta_6-\theta_5$ in Fig. 2). By plotting $\theta-t$ data for the entire run, the 2D trajectories of active shoreline segments can be characterized. Another primary task was to measure the upstream horizontal distances of ABT from the initial source point in centimeters, and to assess whether or not the river is graded.

**R Series**

*Experimental Method and Design*

A 90-cm wide rigid plate with a constant longitudinal slope was placed inside Margi 2. Sediment and water were supplied from the upstream end of a steeper ‘hinterland’ plate, through a sandglass-like funnel (potential error < 2 %) and a tube connected to multiple weirs (potential error < 0.5 %), respectively. They bypassed for some distance over the nonerodible bedrock basement (i.e. the plate surface), and an isolated conical fan delta was built where the feeder streams met the surface of the standing water, i.e. base level. The initial shoreline was located where the two plates were joined.

The subaqueous basement (basin floor) had a constant slope of 0.231. $Q_s$ was 1.178 cm$^3$ s$^{-1}$ including pore, paired with $Q_o$ of 3.81–6.50 cm$^3$ s$^{-1}$. During the entire period of each run, water surface level inside the tank remained constant.

*Experimental Results*

As shown in sequential images of Run R1 (Fig. 3), the delta at the initial stage was built by sheetflow that covered the entire alluvial surface, and sediment was added to the entire deltaic shoreline ($0 = 0–180^\circ$) simultaneously (Fig. 3A). As the alluvial surface progressively expanded, the sheetflow gradually transformed to distributary stream channels, whereby active shoreline segments took scattered positions but incessantly migrated laterally and never stayed at a particular position for a period of more than a few tens of seconds (Figs. 3B, C). Occasionally, only a single, relatively large channel was present on the alluvial plain, but at other times, multiple small distributary channels were present on the delta plain. These two
states of the alluvial feeder system alternated with each other, and this pattern was closely analogous with the observation in the tank experiments of Van Dijk et al. (2009). The experiment was conducted for 2 hours, during which period the delta simply continued to expand not only basinward by progradation but also landward by alluvial aggradation. During the entire run, ABT tended to migrate upstream with minor basinward-landward oscillations.

Figure 4 shows the trajectories of active shoreline segments and ABT obtained from the three runs of the R series. Runs R2 and R3 were conducted with lower and higher water discharges than Run R1, respectively (Table 1). Despite the significant difference in $Q_w$, trajectories of the active shorelines were largely the same as those obtained from Run R1, showing that the active shorelines continued to migrate laterally and occasionally avulsed. The upstream migration of ABT was consistently sustained, as observed in Run R1.

**Interpretation**

Since ABT tended to migrate upstream and did not come to a sustained standstill, alluvial channels on the deltaplain consistently aggraded and never attained grade. While the alluvial system was non-graded with substantial aggradation, the alluvial channels incessantly migrated laterally and often avulsed, and in a relatively short time interval, active channels regressed to previous locations, such that the deltaic shoreline on the whole tended to retain its closely isotropic, arcuate configuration in plan view.

**F Series**

**Experimental Method and Design**

The design of the F series runs to generate an alluvial river at forced grade is basically the same as the R series runs, except that the basinward end of the ‘shelf’ was aligned with a submerged overfall so that the delta could not prograde beyond this line (Figs. 1C, 5). This distinct drop of the basement functioned as a kind of downstream weir against which the alluvial topset river readily attained grade (Cantelli & Muto, 2014). Four runs were conducted under slightly different conditions (Table 1). The shelf was 20 cm in width (dip direction) and had a constant slope of $0.0871\sim0.132$. $Q_s$ was $1.178 \text{ cm}^3 \text{ s}^{-1}$ including pore and was paired with $Q_w$ of $3.81\sim4.63 \text{ cm}^3 \text{ s}^{-1}$. Through out the duration of each run, the water surface level inside the tank remained constant.

Runs of the F series are somewhat analogous to the ones conducted by Kim et al. (2013), for which a delta was built with a vertical rigid plane in the rear such that the delta’s ABT was always fixed on the vertical plane and thus was restricted in horizontal movement. Due to this peculiar topographic setting, it was hard to judge whether the feeder alluvial river became
graded. Apart from consideration of ABT, the behavior of alluvial channels observed in the present runs (see below) was quite similar to that observed in the experiments of Kim et al. (2013).

**Experimental Results**

Figure 5 shows sequential photo images of Run F1. There were no significant differences from the R series runs until the delta toe reached the shelf edge (i.e. overfall), in the respect that (1) the feeder alluvial system developed first as sheetflow and then as distinct stream channels which incessantly migrated laterally and frequently avulsed, (2) ABT tended to migrate landward, and (3) multiple small channels alternated with a single large channel. Soon after the delta toe reached the shelf edge, however, the feeder alluvial system consisted of only a single large channel, or a valley, that was stabilized to the axial area of the delta, the outside of which showed terraced surfaces. The valley had a depth up to 10 mm, several times as deep as the feeder streams. Simultaneously, ABT was migrated slightly basinward and then stopped for an extended period of time. The active feeder system ceased to migrate laterally over the entire alluvial surface but stayed inside the valley, and the entire delta appeared to be stationary despite the continuous supply of water and sediment from upstream (Figs. 5, 6A).

Runs F2–F4 showed differences from Run F1 (Figs. 6B–D). In Run F2, which was conducted with a slightly lower $Q_w$, the feeder alluvial river appeared to be stabilized as was that of Run F1 after the delta toe reached the shelf edge. However, even after this state was attained, the active alluvial system abruptly made a temporary excursion away from the axial position and showed simultaneous multiple active channels. Notable in this episodic event, the active channel subsequently returned to the axial position ($t = 2200–2800$ s). While the active channels deviated from the axial position, ABT moved upstream or downstream to an insignificant extent.

In Run F3, a delta was built on a shelf surface with a gentler slope than in Runs F1 and F2 but with nearly the same $Q_w$ and $Q_s$ as in Run F1 (Table 1). Active channels appear to have terminated migration and avulsion at $t \sim 3000$ s (Fig. 6C), although the delta toe had reached the overfall at $t \sim 2200$ s. Until $t \sim 3000$ s, ABT tended to migrate upstream. Thereafter, ABT moved slightly downstream and remained fixed in position. A characteristic of this run is that even after this point in the run, the active channel still continued to move laterally but within a relatively smaller range (mostly within $80^\circ < \theta < 115^\circ$). Actually, this moment was oscillation rather than migration. When the axial channel was not in the most axial position, ABT was mowing landward.
Run F4, which was conducted with a relatively low $Q_w$, showed behavior analogous to that of the axial channel in Run F3 but in a much more striking manner. Though the feeder alluvial system was stabilized to a relatively wide valley, it continued to show changes in the channel pattern cycling through the following: (1) a single large channel in the axial position, (2) mid-bar aggradation, (3) bifurcation of the axial channel, (4) progressive separation of the channels in transverse directions, and (5) abrupt recurrence of a new single major channel in the axial position. Before the delta toe reached the shelf edge, the feeder channels migrated over the entire delta plain (i.e. $\theta = 0^\circ$–$180^\circ$). When the axial channel was undergoing bifurcation and when the subsequently bifurcated channels moved apart from each other, ABT continued to migrate landward. On the other hand, when the bifurcated channels were converging to form a single large channel, ABT was migrating downstream (Fig. 6D). This cyclic oscillation of ABT was synchronized with the cyclic behavior of the feeder channels: i.e. ABT migrated basinward when a single major channel was being degraded in the axial part and migrated landward when multiple small channels were present and migrating.

In all of the F series runs, distinct terraces formed along the axial channel of the delta plain, resulting from the incision of a valley and the oscillatory behavior of channels accommodated within it. Among these, the formation of terraces at different heights in Run F4 was striking, reflecting the cyclic change of channel pattern (Fig. 7). These appeared to have the characteristic tendencies that (1) higher (i.e. older) terraces have higher slopes, and (2) the terraces extend downstream close to the shoreline (i.e. base level).

**Interpretation**

The sustained standstill of ABT in the late stage of Run F1 (i.e. after the delta toe reached the shelf edge) suggests that the alluvial system was successfully graded. In Run F2, a similar interpretation can hold as for Run F1. A temporary excursion of the main channel from the axial position was autogenic since there was no change in the external conditions ($Q_s, Q_w$). It is noteworthy that the channel autonomously returned to the axial position in a short time, suggesting that the alluvial river recovered grade by itself. On the other hand, alluvial channels in Runs F3 and F4 were not strictly graded, as demonstrated by the oscillation of both the main channel and ABT (Figs. 6C, D). In these two runs, local erosion was evident and sediment accumulated locally even after the delta toe reached the overfall. As for Run F4, a relatively low ratio of $Q_w/Q_s$ can account for local sediment accumulation within the valley that caused the feeder stream to bifurcate. On the other hand, it is not so clear why the feeder stream in Run F3 behaved differently from the one in Run 1 despite having the similar experimental conditions of $Q_s$ and $Q_w$. In general, the channel oscillations in these runs were
limited to a relatively narrow spatial range, and, thus, looked to be stable as a whole, particularly when considered in terms of a relatively long time scale. In this sense, the state attained in Runs F3 and F4 are referred to as ‘quasi-graded.’ It is also noted that the feeder alluvial channel at grade tends to be stabilized in the axial position with insignificant lateral shifting, whereas the feeder channel in an aggradational stage (i.e. before reaching the shelf edge) tends to not remain in a particularly limited zone but frequently migrates laterally and often avulses.

The formation of terraces along the axial channel was a direct consequence of the attainment of grade or quasi-grade. More accurately, it represented the final stage of the river approaching a graded state. As will be discussed later, the recognition of terraces can be a key to identifying graded alluvial rivers in natural systems.

**M Series**

*Experimental Method and Design*

The M series of runs was conducted with no fixed boundary but with a steady fall of base level to incarnate autogenic grade. The basin floor was set to have a uniform basinward inclination, similar to that in all other runs. A necessary condition for the attainment and maintenance of autogenic grade is that the basin floor has the same slope as the alluvial graded river, as noted above. To achieve this setup, preliminary experiments for both Runs M1 and M2 were conducted to measure an alluvial slope at grade for a given set of $Q_s$ and $Q_w$, with the use of a downstream weir (Fig. 1C). The basin slope was then adjusted to be equal to alluvial slope (i.e. $\alpha \sim \phi$). This method of producing a graded river was basically adopted from the experiments of Muto and Swenson (2006), except that they used a 1D flume.

In Run M1, base level dropped at a constant rate of 0.00124 cm s$^{-1}$ from the beginning of the run. This was done using an electromagnetic flow meter that accurately pumped water out at a set discharge. The basin floor slope $\phi$ was 0.0700, and the hinterland slope $\gamma$ was 0.216, both in tangent. The floor of the flume was transversely fluted (to a small depth measured in millimeter) like the teeth of a saw, in order to decelerate the flow. The initial shoreline was located at the intersection between the hinterland basement and the basin floor. A delta was built with constant $Q_w$ (28.3 cm$^3$ s$^{-1}$) and constant $Q_s$ (0.293 cm$^3$ s$^{-1}$), and thus $Q_w/Q_s = 96.6$. Under these conditions, autostratigraphic length scale and time scale (Muto et al., 2007) are given by

$$\Lambda = \frac{Q_s}{R_{str}}$$

$$\tau = 867 \text{ s, respectively, where } \Lambda \text{ and } \tau \text{ are calculated as follows: (1)}$$
\[ \tau = \alpha \frac{\Lambda}{R_{slr}} \]  

The run time of Run M1 (2985 s) at this latter scale was 3.44\(\tau\).

In Run M2, \(\phi = 0.0594\) and \(\gamma = 0.331\). The basin floor was not fluted but was very smooth, in consideration of the foreset height of the delta being significantly small. The initial shoreline was located at the intersection between the hinterland basement and the basin floor. With a constant \(Q_w\) (28.4 cm\(^3\) s\(^{-1}\)) and a constant \(Q_s\) (1.13 cm\(^3\) s\(^{-1}\)), a delta was built having a \(Q_w/Q_s = 25.1\), which was much lower than that in Run M1. Base level was continuously lowered at a constant rate of 0.00353 cm s\(^{-1}\) by mechanical pumps connected with weirs. Under these conditions, autostratigraphic scales are \(\Lambda = 17.9\) cm and \(\tau = 301\) s. Owing to the small value of \(\tau\) and the use of a large tank, Margi 4, the alluvial system could be observed for a considerably long time (6570 s, i.e. 21.8\(\tau\)).

**Experimental Result**

The sequential images of the formation of deltas in Runs M1 and M2 are shown in Fig. 8 and 9, respectively. In the initial stage of Run M1, the alluvial feeder system had no distinct stream channels but developed a single deltaic lobe by sheetflow (Fig. 8A). As the delta grew, sheetflow began to branch off and be localized to develop discrete channel-lobe units (Figs. 8B–E). Those channel-lobe units seldom migrated in transverse directions and never avulsed but simply extended basinward in a linear trend. Reflecting this basinward extension, active shoreline segments progressively decreased in azimuth interval as they migrated basinward (Fig. 10A). During the run, ABT consistently migrated basinward at a decelerating rate. The net migration distance of ABT measured at the end of the run was 7.5 cm (0.49\(\Lambda\)), whereas a major lobe extended basinward with a length of 111.5 cm (7.24\(\Lambda\)).

The pattern observed in Run M2 (Fig. 9) was basically the same as that in Run M1 with respect to the basinward linear extension of an active channel-lobe unit, but the distance traveled was much longer (386 cm, i.e. 21.6\(\Lambda\)) than that in Run M1 (Fig. 10B). In Run M2, ABT migrated landward at an early stage with a decelerating rate and came close to stopping in the late stage. The landward migration distance of ABT (6.2 cm) was only 0.35\(\Lambda\), which was negligibly small when compared to the basinward travel distance of the channel-lobe unit. Figure 10A shows the dimensionless trajectories of active shoreline segments and ABT, both given in a linear scale instead of azimuth angles. Trajectories obtained from Run M1 showed considerable overlap with trajectories from the early part of Run M2.

**Interpretation**
The eventual standstill of ABT in Run M2 implies that the feeder alluvial river had attained grade. In contrast, the alluvial system in Run M1 failed to attain grade but did attain quasi-grade, and ABT did not stop migrating but was still transient toward the attainment of grade, even at the end of the run (Fig. 11B). The linear-scale, dimensionless shoreline trajectory diagram (Fig. 11A) suggests that the alluvial system in Run M1 represented only the very early phase of the process of approaching a graded state, whereas the almost entire period of the approach to grade was captured in Run M2. Figure 11B also implies that it requires ~10τ for the channel-lobe system to attain a graded state that is characterized with the halt of ABT and a constant width of the deposit.

The alluvial system in Run M2 was under an aggradational regime in the early stage, as suggested from the landward migration of ABT. This is contrary to Run M1, where ABT migrated basinward. This striking difference perhaps arose from the significant differences in magnitude of $Q_w/Q_s$.

ATTAINMENT OF ALLUVIAL GRADE
The results of the present 2D experiments clarify that an alluvial topset river of a delta can attain grade (or quasi-grade) in a setting where the river streams are free from lateral confinement. Attainment of this 2D grade is possible by at least two different mechanisms: (1) equilibrium response of a delta to stationary base level in a downstream-fixed boundary setting (forced grade in the F series), and (2) nonequilibrium response of a delta to constant base-level fall in a moving boundary setting where the basin floor has the same slope as the alluvial topset (autogenic grade in the M series). In stark contrast, a feeder alluvial river never becomes graded in a moving-boundary system with stationary base level (the R series). The results of the present 2D experiments are consistent with the existing 1D models (Muto & Swenson, 2005, 2006) and thus are critical to the conventional notion that alluvial grade in a prograding deltaic system is attained with stationary base level (e.g. Posamentier et al., 1988; Galloway, 1989; Shanley & McCabe, 1994).

Under a downstream-fixed condition, alluvial grade can be attained with stationary base level (the F series). At the very moment that grade is attained, however, there inevitably occurs valley incision, i.e. a significant erosion of existing delta deposits. The resulting graded streams run through the valley outside of which there are paired stream terraces and an abandoned delta plain surface. This view of alluvial grade is significantly different from the one that has been incorporated in conventional sequence stratigraphy, and raises doubts with respect to the conceptual validity and physical significance of subaerial accommodation.
INHERENT SPATIAL STABILITY OF THE GRADED SYSTEM

Another important finding in the present study is the stabilization inherent to a graded alluvial channel (or channel-lobe unit), as shown by the runs in the F and M series. It appears that autocyclic transverse shifting of active alluvial channels is suppressed in the graded system regardless of whether the downstream boundary is fixed. A primary mechanism of channel stabilization is understood in terms of analyzing what happens in an aggradational alluvial system. In an aggrading alluvial system (the R series), the global topography continuously changes in association with channel migration and avulsion, i.e. local erosion and deposition under the regime of progressive burial. Local sediment accumulation along channels, accompanied by modification of local transverse gradients, functions to force the active channel to change its flow path, which then gives rise to new local events of erosion and simultaneous or subsequent deposition (Van Dijk et al., 2009). With this autocyclic feedback process, active channels consistently change their positions with occasional avulsions, such that after a relatively long lapse of time, the alluvial system can produce a uniform depositional surface as a whole. This process is also called ‘compensational stacking’ (Straub et al., 2009). Once a feeder channel has attained grade (the F series), on the other hand, global aggradation no longer occurs, though local accumulation and erosion are still possible as implied from Run F4. Even in this latter case, however, the autocyclic behavior of feeder channels does not prevail over the entire alluvial system but is suppressed to a limited extent along the axial part of the entire system.

Effects of Basin Water Depth on Distributary Channel Behavior

A series of experimental runs conducted by Muto et al. (2011), here referred to as the BW series, provides a rationale for channel stabilization as the effect of basin water depth. In their experiments, a delta was built on a differential basement, which was created by placing a platform on the flat bottom surface so that local basin water depth abruptly changed in the direction transverse to the progradation of the delta (Fig. 12). Their experimental results clearly indicate that basin water depth affects the dynamics and autocyclic behavior of distributary channels in the delta plain. Where a channel empties to deeper water, it selectively stays there for a longer period of time and migrates more slowly than where it empties to shallow water. Owing to this response of distributary channels to local basin water depth, the delta evolves in a way such that it retains an isotropic shoreline configuration despite the differential water depths. If basin water in front is extremely deep (such that the delta cannot prograde), the feeder channels will be stabilized in disassociation with autocyclic
lateral shifting. Following this novel notion from the Bw series runs, a theoretical model is here developed for characteristic channel behavior reflecting the effect of basin water depth.

For simplification, a delta plain is assumed to have a vertical wall behind ($\gamma = 90^\circ$) and has a central angle of $\lambda$ in radians (Fig. 13), as adopted in the Bw series runs. Base level remains stationary, and the basin floor is horizontal ($\phi = 0^\circ$) with a constant water depth of $h$. If the delta retains the overall geometry specified with angle parameters during period $t$ of progradation from the onset of the delta growth, the following equation of mass balance holds true:

$$\frac{\alpha \lambda}{6} x^3 + \frac{\lambda}{2} hx^2 + \frac{\lambda}{2\beta} h^2 x + \frac{\lambda}{6\beta^2} h^3 = Q_s t$$

(3)

where $x$ is delta plain radius, $\alpha$ is mean alluvial topset slope in tangent and $\beta$ is mean subaqueous foreset slope in tangent. The first term on the left side represents sediment volume of the alluvial topset (the deposit lying above base level); the rest represents the subaqueous part of the delta (the deposit lying below base level). Differentiating this equation with respect to $t$, an overall progradation rate $R_{\text{pro}}$ and an overall alluvial aggradation rate $R_{\text{agg}}$ are found:

$$R_{\text{pro}} = \frac{2\beta Q_s}{\lambda(\alpha \beta x^2 + 2\beta hx + h^2)}$$

(4)

$$R_{\text{agg}} = \frac{2\alpha \beta Q_s}{\lambda(\alpha \beta x^2 + 2\beta hx + h^2)}$$

(5)

In the case that $h$ is negligibly small compared with $\sqrt{\alpha \beta x}$, i.e. the basin water is extremely shallow, the corresponding values of $R_{\text{pro}}$ and $R_{\text{agg}}$ are given by:

$$R_{\text{pro}}^{h \rightarrow 0} = \frac{2Q_s}{\alpha \lambda x^2}$$

(6)

$$R_{\text{agg}}^{h \rightarrow 0} = \frac{2Q_s}{\lambda x^2}$$

(7)

Equations 4 and 5 can be made dimensionless by dividing with Eqs. 6 and 7, respectively:

$$R_{\text{pro}}^* = \frac{R_{\text{pro}}}{R_{\text{pro}^{h \rightarrow 0}}} = \frac{1}{1 + 2h_* + \alpha_* h_*^2}$$

(8)

$$R_{\text{agg}}^* = \frac{R_{\text{agg}}}{R_{\text{agg}^{h \rightarrow 0}}} = \frac{1}{1 + 2h_* + \alpha_* h_*^2}$$

(9)

where $h_*$ is dimensionless basin water depth and $\alpha_*$ is alluvial slope normalized with $\beta$, each defined as:

$$h_* = \frac{h}{\alpha x}$$

(10)
\[ \alpha_s = \frac{\alpha}{\beta} \]  

(11)

As is evident from Eqs. 8 and 9, the rates of shoreline-averaged delta progradation and overall alluvial aggradation are both markedly affected by basin water depth. To account for this, grade index \( G_{\text{index}} \) is here defined to quantify the effect of basin water depth, to encompass \( R_{\text{pro}}^* \) and \( R_{\text{agg}}^* \) and also to denote how closely the feeder alluvial system is graded:

\[ G_{\text{index}} = \frac{1}{1 + 2h_* + \alpha_s h_*^2} \]  

(12)

Clearly, \( G_{\text{index}} = 0 \) for grade (i.e. perfect bypass of the supplied sediment to the delta’s subaqueous slope) corresponding with \( R_{\text{pro}}^* = R_{\text{agg}}^* = 0 \), and \( G_{\text{index}} = 1 \) for perfect alluvial aggradation (i.e. no sediment accumulation basinward of the shoreline) corresponding with \( R_{\text{pro}}^* = R_{\text{agg}}^* = 1 \). Equation 12 suggests that the alluvial topset river of the delta approaches grade as \( h_* \) increases toward \(+\infty\), and perfect aggradation is achieved as \( h_* \) decreases toward 0.

Another rate to be considered is that of mean channel migration along the shoreline. Delta distributary channels can change location by two different modes: channel shifting and avulsion. The time scale of alluvial river avulsion \( \tau_A \) can be approximated as the time interval required for the river to aggrade one average channel depth \( h_c \) (Mohrig et al., 2000; Jerolmack & Mohrig, 2007; Reitz et al. 2010), such as for:

\[ \tau_A = \frac{h_c}{R_{\text{agg}}} \]  

(13)

Avulsion frequency of the river system \( f_A \) is then given by:

\[ f_A = \frac{N}{\tau_A} = \frac{NR_{\text{agg}}}{h_c} \]  

(14)

where \( N \) is the number of active channels. In the case that \( h \sim 0 \), the corresponding values are given by:

\[ \tau_{A h \sim 0} = \frac{h_c}{R_{\text{agg}} h \sim 0} = \frac{\lambda h_c x^2}{2Q_s} \]  

(15)

\[ f_{A h \sim 0} = \frac{N}{\tau_{A h \sim 0}} = \frac{2NQ_s}{\lambda h_c x^2} \]  

(16)

Equations 13 and 14 can be made dimensionless by dividing with Eqs. 15 and 16, respectively, to yield:

\[ \tau_{A^*} = \frac{\tau_A}{\tau_{A h \sim 0}} = \frac{R_{\text{agg}} h \sim 0}{R_{\text{agg}}} = R_{\text{agg}}^{* -1} = G_{\text{index}}^{-1} \]  

(17)
\[ f_{A^*} = \frac{f_A}{f_{A^*}} = R_{agg^*} = G_{\text{index}} \]  \hspace{1cm} (18)

Jerolmack & Mohrig (2007) also suggest a time scale for channel shifting \( \tau_s \), given by:

\[ \tau_s = \frac{B}{\nu_s} \]  \hspace{1cm} (19)

where \( B \) is average width of the active channels and \( \nu_s \) is bank erosion rate, which, if measured along the shoreline, is approximately equal to the migration rate of active shoreline \( R_{\text{mig}} \):

\[ R_{\text{mig}} \sim \nu_s \]  \hspace{1cm} (20)

Lateral shifting of distributary channels occurs because of a local decrease of channel slope (Olariu & Bhattacharya, 2006; Straub, 2009), which can arise from the basinward extension of the channel mouth, i.e. shoreline progradation. Thus, \( R_{\text{pro}} \) will positively correlate to \( R_{\text{mig}} \), implying the following relationship:

\[ R_{\text{mig}} = \Omega R_{\text{pro}} = \alpha^{-1} \Omega R_{agg} = \alpha^{-1} \Omega R_{agg,h=0} G_{\text{index}} \]  \hspace{1cm} (21)

where \( \Omega \) is the dimensionless positive coefficient that will be related to parameters including grain-size distribution or vegetation of flood plains. In fact, the experimental data of Run Bwl supports the linear correlation between \( R_{\text{mig}} \) and \( R_{agg} \) (Fig. 14), where \( \Omega \) is on the order of \( 10^1 \) for the given experimental conditions: i.e. \( R_{\text{mig}} \sim 10^1 R_{\text{pro}} \sim 10^2 R_{agg} \). If this linear relationship holds true even in case \( h \sim 0 \), dimensionless rate of channel migration along shoreline is given by:

\[ R_{\text{mig}^*} = \frac{R_{\text{mig}}}{R_{\text{mig},h=0}} G_{\text{index}} \]  \hspace{1cm} (22)

where

\[ R_{\text{mig},h=0} = \Omega R_{\text{pro},h=0} = \alpha^{-1} \Omega R_{agg,h=0} \]  \hspace{1cm} (23)

On the other hand, the time scale of channel shifting (Eq. 19) can be rewritten as:

\[ \tau_s = \frac{B}{\nu_c} = \frac{B}{\Omega R_{\text{pro}}} = \frac{\alpha B}{\Omega R_{agg}} \]  \hspace{1cm} (24)

The dimensionless time scale of channel shifting \( \tau_s^* \) is thus given as follows:

\[ \tau_s^* = \tau_{s,h=0} \tau_s = \frac{R_{agg,h=0}}{R_{agg}} = R_{agg}^{-1} = G_{\text{index}}^{-1} \]  \hspace{1cm} (25)

It may also make sense to consider time scale of channel shifting \( \tau_r \) reflecting the mean recurrence period of a particular channel to a particular position:
\[
\tau_r = \frac{\lambda x}{R_{\text{mig}}} = \frac{\alpha B \lambda x}{\Omega R_{\text{agg}}}
\]  
(26)

Dimensionless recurrence time is thus given by:

\[
\tau_{r*} = \frac{\tau_r}{\tau_{r*0}} = \frac{R_{\text{agg*}}}{R_{\text{agg}}} = G_{\text{index}}^{-1}
\]  
(27)

In summary, the following simple relation is suggested:

\[
R_{\text{pro*}} = R_{\text{agg*}} = R_{\text{mig*}} = f_{\text{A*}} = \tau_{\text{A*}}^{-1} = \tau_{\text{s*}}^{-1} = \tau_{\text{r*}}^{-1} = G_{\text{index}}
\]  
(28)

If the delta is unable to prograde basinward \((R_{\text{pro*}} \sim 0)\) because of very deep basin water in front \((h* \gg 1, \text{virtually } +\infty)\), then the delta plain does not aggrade \((R_{\text{agg*}} \sim 0); \text{i.e. the attainment of grade}\), the alluvial topset river is stabilized \((R_{\text{mig*}} \sim 0)\) in association with very long time scales \((\tau_{\text{A*}} = \tau_{\text{s*}} = \tau_{\text{r*}} \sim +\infty, \text{virtually})\). The stabilization of the alluvial feeder system \((R_{\text{mig*}} \sim 0)\) inevitably accompanies the attainment of alluvial grade as a consequence of mass balance sustained under the condition that the delta retains its geometrical aspects \((\alpha, \beta, \gamma, \phi, \lambda \text{ and isotropic shoreline configuration})\).

Note that grade index has no time component and thus can be valid at any time point as far as the angle parameters are not changed. Valley incision that occurs at the attainment of grade is the moment of a significant change in the delta geometry (particularly, abrupt decrease in \(\alpha\) and \(\lambda\)). After the incision is complete, however, the model is again applicable to the deltaic system that is now confined within the valley but with new values of \(\alpha\) and \(\lambda\).

Grade index may be applied to both the entire system and a sector segment of a delta. In the case of Run B2 (Fig. 12), for example, \(G_{\text{index}} = 0.888\) for shallow water, 0.414 for deep water, and 0.510 for averaged water depth at the end of run (see Table 2 for other runs).

**Geometrical Modeling for the F Series Runs**

The synthetic view of \(R_{\text{pro*}}, R_{\text{agg*}}, R_{\text{mig*}}, f_{\text{A*}}, \tau_{\text{A*}}, \tau_{\text{s*}}, \tau_{\text{r*}}\) in terms of the control of basin water depth (Eqs. 28) can partly account for what happened in the F series runs. However, the topographic setting for the F series runs is significantly different from the one that is assumed for the equations suggested above from the point-of-view that (1) the hinterland basement is not vertical but gently inclined \((\gamma \sim 0.45 \text{ in tangent})\), (2) the basin floor landward of the overfall is not horizontal but tilting basinward at a slope of \(\phi \sim 0.1 \text{ in tangent})\), and thus (3) basin water depth in front of a delta is not spatially uniform but increases basinward, proportional to \(x\):
With these slope conditions, dimensionless basin water depth in front of a delta takes a constant value that is determined only with the geometrical parameters:

\[ h = \frac{\beta \phi}{\beta - \phi} x \]  \hspace{1cm} (29)

For the topographic conditions adopted in the F series runs, Eq. 3 is modified as:

\[ \frac{\alpha \gamma \pi}{6(\gamma - \alpha)} x^3 + \int_0^h S(\eta) d\eta = Q_t \]  \hspace{1cm} (31)

where \( \eta \) is water depth of a foreset surface (0 ≤ \( \eta \) ≤ \( h \)) and \( S(\eta) \) is given by:

\[ S(\eta) = \left[ x + \frac{\eta}{\beta} \right]^2 \arccos \left( \frac{\beta \eta}{\beta \phi x + \phi \eta} \right) - \frac{\eta}{\phi} \left( x + \frac{\eta}{\beta} \right) \sqrt{(\beta \phi x + \phi \eta)^2 - \beta^2 \eta^2} \]  \hspace{1cm} (32)

The first and second terms on the left side of Eq. 31 represent the sediment volume of the subaerial and subaqueous parts of the delta, respectively. It is difficult to find an analytical solution to this equation, but a numerical solution can be found by making some assumptions. Because of the topographic conditions assumed, the whole deltaic deposit does not change in shape with time, as reflected in the constant ratio of \( x \) to \( h \) (Eq. 29). Thus, sediment volume ratio \( \kappa \) of the subaqueous part to the subaerial part is kept constant throughout progradation of the delta, which is easily confirmed through numerical simulation. Equation 31 can therefore be reduced to:

\[ \frac{\alpha \gamma \pi}{6(\gamma - \alpha)} (1 + \kappa) x^3 = Q_t \]  \hspace{1cm} (33)

From this simple expression, \( R_{\text{pro*}} \), \( R_{\text{agg*}} \) and \( R_{\text{mig*}} \) are found through a similar procedure as considered for the Bw series runs:

\[ R_{\text{pro*}} = R_{\text{agg*}} = R_{\text{mig*}} = f_{A*} = \frac{1}{1 + \kappa} \]  \hspace{1cm} (34)

Grade index applicable to the F series runs can thus be defined as:

\[ G_{\text{index}} = \frac{1}{1 + \kappa} \]  \hspace{1cm} (35)

Given a finite value of \( \kappa \) (0 < \( \kappa < +\infty \)), \( G_{\text{index}} \) takes a constant value between 0 and 1. In the case of the F series runs, \( h_* = 0.55-0.75 \), \( \kappa = 0.42-0.54 \), whereby \( G_{\text{index}} = 0.64-0.70 \). Note that these are valid before the delta toe reaches the overfall. After this has happened, however, \( \phi \) abruptly exceeds \( \beta \) and thus \( h_* \) is virtually +\( \infty \) (i.e. \( G_{\text{index}} \sim 0 \)).

The models suggested above for Bw series runs and F series runs are not applicable to M series runs, where base level was not stationary and the delta’s overall geometry significantly
changed with time, as is indicated by continuous decrease in $\lambda$. In the M series (especially Run M2), alluvial grade was attained owing to delta progradation ($R_{pro} = \text{const} > 0$) and constant base-level fall ($R_{slf} = \text{const} > 0$). Despite this fundamental difference, however, graded alluvial channels observed in the M series show basically the same behavior as in the F series in the respect that they did not migrate significantly in transverse directions and thus behaved in a non-compensational manner described as:

\[ R_{agg} \sim 0 \Leftrightarrow R_{mig} \sim 0 \text{ (but } R_{pro} > 0) \]  

Because of the moving boundary condition, sediment accreted on the delta’s foreset slope allowed the active channel(s) to extend basinward, whereby a far-basinward protruding lobe formed, leaving ABT far landward behind.

SEARCH FOR GRADED ALLUVIAL RIVERS IN NATURAL SYSTEMS

The present experimental study implies that an alluvial river feeding a delta can be graded in the following two scenarios: (1) with stationary base level, the delta develops a bypass system and transports all sediment into deep water ($h^* >> 1$) and cannot prograde (i.e. forced grade), and (2) with constant base-level fall, the delta progrades above a preformed alluvial surface that extends basinward during a preceding base-level fall and then drowns during a subsequent base level rise (i.e. autogenic grade). Following these implications, natural environments in which graded alluvial rivers are likely to be found are predicted.

Modern Fan Deltas in Front of Deep Water

An alluvial river at forced grade would most likely be found in modern fan deltas that have developed in front of deep water ($h^* >> 1$), such that the delta cannot prograde basinward ($R_{pro} \sim 0$). A criterion to identify graded alluvial systems suggested from the F series runs is the presence of river terraces along an axial channel/valley that was incised into the delta plain of the previous stage by the process of approaching grade.

Incised stream channels and their adjacent terraces have usually been attributed to graded versus nongraded states of the rivers (Ritter, 1967; Bull, 1979, 1991; van Heijst and Postma, 2001) and ultimately to changes in rates or types of external dynamic forcing, such as tectonic uplift, eustatic fluctuation of sea level, or climate change affecting precipitation and detritus productivity in the watershed (Bull, 1991; Veldkamp, 1992). In addition to these perturbations in the environment that produce instability in the fluvial system (Merritts et al., 1994; Hancock & Anderson, 2002), it is also known that river deltas show repeated incision and multiple terrace formations with constant base-level fall (Muto & Steel, 2004). Those autogenic terrace surfaces basically have the same slope as the feeder alluvial river running
through between terraces. On the other hand, autogenic terraces produced with stationary base level in the F series runs had higher slopes than the feeder channel; thus, a higher (older) terrace has a higher slope, and a lower (younger) terrace has a lower slope (Fig. 7). This can be a criterion for identifying graded alluvial systems that were produced under downstream-fixed boundary settings.

Fan deltas which appear to illustrate this latter type of autogenic terraces are found on the Japan Sea coast of central Japan, located to the south of Toyama Bay, where the Toyama Deep-Sea Canyon (TDSC) extends close to the coastline (Nakajima, 2006; Fig. 15). The southern landward margin of Toyama Bay is steep (15’), high in relief (up to 1000 m) and fringed by a very narrow (0–4 km wide) shelf. Several small tributary canyons of TDSC are deeply incised in the bay’s marginal slope with head segments extending to the mouths of the rivers that have fed sediment to the fan deltas. Immediately landward the fan deltas are the Northern Japan Alps, a range including the highest mountains in Japan with peaks rising 3000 m above sea level.

Figure 16 shows longitudinal profiles of terraces along the axial channels of the Kurobegawa, the Katakaiigawa, the Hayatsukigawa and the Joganjigawa (gawa = river in Japanese). Among them, the Joganjigawa fan is presently separated from the coastline, but it is simply assumed that the river, or an ancestral one, was previously building a fan delta. The longitudinal profile of each river has a nearly uniform downstream slope and thus is approximated with a straight line, with an exception for the lowermost reaches of the Joganjigawa. Mean alluvial slopes (α) vary significantly, ranging between 0.0090–0.0197 for the present river beds and between 0.011–0.0204 for the youngest terraces (< 10 kaBP). Ages of the terraces are also varied; for example, the oldest terrace of the Joganjigawa formed at 45 –55 kaBP, whereas no terrace of this age range is present in the Kurobegawa where the oldest terrace formed at 20–30 kaBP. Nevertheless, there is a clear tendency that in each of the rivers, (1) the present-age profile has a gentler slope than the terraces and (2) higher (i.e. older) terraces have steeper slopes (Fig. 16).

Neotectonic movements in the Toyama Bay region could account for the tendency of the progressive decrease in terrace slopes. In fact, this region has subsided as much as 10 cm for ca. 100 years from 1883–1913 to 1986–2000 relative to the Northern Japan Alps region, which has also subsided but at a smaller magnitude (Geographical Survey Institute, 2002). However, it is uncertain if this subsidence trend has persisted for a much longer period (e.g. 10^5 years) and also if the coastal region has been undergoing a tectonic tilt basinward. If the progressive decrease in terrace slopes were completely due to the cumulative tectonic tilting of the region, the rate of tilting would have been significantly different by rivers; e.g. 0.0092°
−0.0138° ka\(^{-1}\) in the Joganjigawa, but 0.0160°−0.0235° ka\(^{-1}\) in the Kurobegawa. An alternative or secondary mechanism is the alluvial process of approaching grade, which is shown by progressive decrease in \(G_{\text{index}}\). Values of \(G_{\text{index}}\) can be calculated with Eq. 13 on the assumption that (1) tectonic tilting is not significant and (2) \(h \sim 1000\) m and \(\beta \sim 0.27\) (15°) (ref. Nakajima, 2006). In the case of the Kurobegawa fan delta (\(x = 13.3\) km), for example, \(G_{\text{index}}\) decreased from 0.003 at 20–30 kaBP to 0.0005 at present, implying that the Kurobegawa has been getting closer to grade during the past 20–30 ka. A similar trend of \(G_{\text{index}}\) is shown for the other three rivers (Fig. 16). Possibly, tectonic tilting and the river process of approaching grade proceeded in some combination.

The ages of the terraces in the Toyama Bay coast region are up to over 100 kaBP (Fig. 16), and eustatic sea level has since changed with an amplitude of over 100 m. However, even with this magnitude of eustatic fluctuation, there were no significant changes in the topographic setting in which the deltaic systems were located in front of the deep water at least during the past \(10^5\) years. Provided a set of assumptions, avulsion time scale for the river systems can be calculated with Eqs. 15 and 17, though Eq. 15 is slightly modified to include intermittency \(I\) of flooding events that have an effect on the transport of sediment:

\[
\tau_{A=0} = \frac{\lambda h c x^2}{q_s A I} \tag{37}
\]

where \(q_s\) is sediment yield intensity and \(A\) is drainage area, giving \(Q_s = q_s A\). In the case of the Kurobegawa at 20–30 kaBP, \(\lambda \sim \pi / 4\), \(q_s = 1.929 \times 10^3\) m\(^3\) km\(^{-2}\) year\(^{-1}\) and \(A = 667\) km\(^2\) (Kobayashi & Tanaka, 2012). In addition, \(I\) is assumed to around 0.05, because major flooding events with a discharge of 3100−6700 m\(^3\) sec\(^{-1}\) occurred at intervals of ca. 20 years over the past 100 years (Ministry of Land, Infrastructure, Transport and Tourism, 2006). The recorded flood water depths were around 3 m, for which we assume that \(h_c \sim 3\) m. With these numerical values, \(\tau_A \sim 1.0 \times 10^6\) years, implying that it required a much longer time for the river to approach or attain grade than in a single cycle of eustatic changes (Table 3).

**Fossil Deltas in Outer to Marginal Shelves**

An attempt to seek out autogenically-graded alluvial rivers in natural systems, whether modern or ancient, is in progress, though no clear example has been yet identified. For the attainment of autogenic grade under a moving boundary condition, it is required that (1) sea level (i.e. base level) falls at a constant rate and (2) basin slope \(\phi\) coincides with alluvial slope \(\alpha\). This geometrical condition can be realized in outer to marginal areas of the deltaic shelf which were originally alluvial plains but were then submerged during in subsequent stages of sea-level rise, as explained by the extended autoretreat model (Tomer et al., 2011).
A river emptying into sea has a critical alluvial length \( L_{\text{crt}} \) over which the river cannot maintain deltaic sedimentation during sea-level rise. \( L_{\text{crt}} \) is determined by:

\[
L_{\text{crt}} = \frac{\gamma \sqrt{1 + \alpha^2}}{\gamma - \alpha} \Lambda - \Lambda \quad \text{(if } \alpha << \gamma) \tag{38}
\]

In the case that there is no pre-existing deposit on the shelf surface (i.e. alluvial length \( L = 0 \) \( << L_{\text{crt}} \)) at the onset of sea level rise, regression occurs in the early stage of sea level rise, which is then followed by deltaic transgression and eventually by nondeltaic transgression. This critical event is referred to as ‘autobreak’ for \( \gamma < \beta \) (Muto, 2001) or ‘autodrowning’ for \( \gamma > \beta \) (Tomer et al., 2011). The occurrence of an autobreak event is coincident at the time when \( L \) becomes equal to \( L_{\text{crt}} \). If the alluvial river has extended seaward over a significantly long distance during sea level standstill (or lowering) prior to the onset of sea level rise, the depositional system undergoes rapid transgression as soon as sea level starts to rise, whereby a critical event, which looks like autobreak, occurs, although this event is not autogenic. Thereafter, sediment supplied is entirely consumed for the feeder alluvial river to aggrade and is no longer able to accumulate beyond the shoreline (i.e. \( G_{\text{index}} \sim 1 \)). Then, the shoreline, which is now nondeltaic, rapidly recedes landward, leaving a concave-upward surface, the downstream part of which can have the same slope as \( \alpha \).

During the subsequent cycle of sea level fall, a new delta can have a chance to prograde on a submarine floor that has a slope of \( \alpha \), which favors the feeder alluvial river to attain grade. The stratigraphic sign of ancient graded rivers, if any, might well be found in the strata of the outer to marginal part of broad shelves that accumulate during sea level fall. Such deltas would lack topset strata and significant fluvial erosional features and would also be characterized by the development of channel-lobes that have basinward-protrusions in planometric geometry. Thus, a thorough investigation of outer to marginal shelves is expected to result in the identification of ancient graded rivers.

**CONCLUSIONS**

1. The present tank experiments clarify that an alluvial river feeding a delta can become graded in a 2D setting where alluvial activities are not confined by unerodible basement topography in transverse directions. The 2D alluvial grade can be attained through at least two different ways: (1) ‘forced grade’ attained by equilibrium response to stationary base level in a downstream-fixed boundary setting, and (2) ‘autogenic grade’ attained by nonequilibrium response to a constant base-level fall in a moving-boundary setting.

2. Autocyclic lateral shifting and avulsion of river channels tend to be intensely suppressed in a graded alluvial system, whether the state of grade attained is forced or autogenic. A
graded alluvial channel can be stabilized to a particular location of the delta plain (e.g. axial part) and seldom move apart from that position. The feeder alluvial river at forced grade is located within a valley that incises the delta plain, whereby the outside surface of the channel becomes pared stream terraces. Under a moving boundary condition, on the other hand, the feeder river at autogenic grade simply extends basinward without valley incision and lateral shifting. A delta lobe fed by this type of graded river can have strikingly basinward-protruding geometry showing a linear trend.

3. Basin water depth in front of a delta affects the dynamics and time scales of distributary channels on the delta plain. As basin water depth increases, channel migration slows and avulsion occurs less frequently. If basin water is extremely deep, neither migration nor aggradation occurs. Forced grade attained under a downstream-fixed condition represents the latter case. This synthetic view, described with grade index, encompasses (1) delta progradation, (2) alluvial aggradation and grade, (3) channel shifting and stabilization, and (4) time scales of channel behavior, accounting for what happened in the runs of the F series.

4. Natural environments that favor the attainment of alluvial grade include (1) modern fan deltas developing in front of very deep water (forced grade) and (2) ancient deltas prograding over outer to marginal shelves with falling sea level (autogenic grade). Searches for recent, subrecent or ancient graded rivers will most likely be successful in these places.

ACKNOWLEDGMENTS
This work was financially supported in part by a Japanese Grant-in-Aid for Scientific Research (B2034140, C25409489) to TM.

NOMENCLATURE

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**REFERENCE**


Muto, T., Miao, H. and Parker, G. (2011) How do deltas respond as they prograde over bathymetry that varies in the transverse direction?: Results of tank experiments. *RCEM*


Veldkamp, A.A. (1992) A3-D model of Quaternary terrace development, simulations of terrace stratigraphy and valley asymmetry: A case study for the Allier terraces (Limagne, France), Earth Surface Proc. Landforms, 17, p. 487–500. Table and Figure captions
Table 1. Experimental conditions and results for runs in the R series, the F series and the M series.

Table 2. Application of the geometrical model to B_W series run data reported by Muto et al. (2011).

Table 3. Dataset of the Kurobegawa.

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Figure 6. Trajectory diagrams of active shoreline segments (left column) and ABT (right column) obtained from F series runs. In each run, (1) the behavior of active shoreline segments changed drastically soon after the delta toe had reached the overfall, and (2) ABT simultaneously came to a halt after insignificant basinward migration.

Figure 7. An image of the delta built in Run F4 taken well after the delta toe had reached the overfall. Note the presence of well-developed paired terraces along the axial channel. There is a clear tendency that (1) higher (i.e. older) terraces had higher slopes, and thus (2) the present active river bed has the lowest slopes at any time in the evolution of the alluvial system. These terraces are autogenic since the dynamic forcing ($Q_w, Q_s$, base level) was kept constant during the run.

Figure 8. Sequential photo images of Run M1, where a delta was built with constant base-level fall. Note that a particular delta lobe extended linearly basinward without any significant lateral shifting.

Figure 9. Sequential photo images of Run M2, where a deltaic lobe extended over 3 m basinward with constant base-level fall as in Run M1, but in a larger tank.

Figure 10. Trajectory diagrams of active shoreline segments (left) and ABT (right) obtained from M series runs. In each run, (1) the lateral extent of migration of active shoreline segments progressively converged to the axial position ($\theta \sim 90^\circ$), and (2) the landward migration of ABT decelerated and came to a stop.

Figure 11. Dimensionless linear-scale trajectory diagrams of active shoreline segments (left) and ABT (right) obtained from M series runs. It appears that the alluvial system observed in Run M1 represented only a significant early stage of its evolution toward the graded state.

Figure 12. River deltas built up in different runs of tank experiments conducted by Muto et al. (2011), which are referred to as the ‘$B_W$ series’ in the present paper. The objective of this series was to clarify the control of local basin water depth on the growth of the deltaic body. For this objective, a differential basement was created by placing a platform on the horizontal bottom surface. Basin water depth was 7.2 cm on the platform and 0.7 cm elsewhere. Run $B_W1$ was the reference run of this series, which was conducted with a uniform basin water depth of 7.2 cm. In each of Runs $B_W1$–4, the maximum thickness of the final alluvial deposit
was no more than 4 cm; thus, the differential water depth must have exerted a marked influence on delta progradation. Nevertheless, the resulting shoreline configurations look quasi-isotropic, implying that differential bathymetry exerted no critical influence on the overall shoreline configuration. Light concentric lines are projections of horizontal laser lines which were at a vertical interval of 1.0 cm. For detailed experimental conditions, see Muto et al. (2011).

Figure 13. Schematic illustration of a delta assumed in geometrical modeling. See text for a detailed explanation.

Figure 14. Linear relationship between $R_{\text{mig}}$ and $R_{\text{agg}}$ for measurements from Run B1. $R_{\text{mig}}$ values were adopted from Muto et al. (2011), and $R_{\text{agg}}$ values were provided by calculation with other parameters including grade index.

Figure 15. Topographic setting of fan deltas/alluvial fans on the Japan Sea coast of central Japan. Note that most of the feeder river mouths have direct connections with submarine canyons of the Toyama Deep Sea Channel. J = Joganjigawa, H = Hayatsukigawa, K = Katakaigawa, Kr = Kurobegawa. Simplified from Nakajima et al. (1998).

Figure 16. Downstream profiles of terraces and present river beds. Numerical values for alluvial slopes ($\alpha$) and grade indexes ($G_{\text{index}}$) are provided. Note that in each river system higher terraces have higher slopes. Ages of the terraces are based on chronological data by Matsuura et al. (2007) and Fujii et al. (2011).
Table 1. Experimental conditions and results for runs in the R series, the F series and the M series.

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<th>Run</th>
<th>Boundary condition</th>
<th>R1</th>
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<th>F2</th>
<th>F3</th>
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Notes:
- Values at the end of run, averaged over the entire shoreline.
### Table 3. Dataset of the Kurobegawa

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MLIT: Ministry of Land, Infrastructure, Transport and Turism
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Figure 4. Trajectory diagrams of active shoreline segments (left column) and ABT (right column) obtained from R series of runs. Note that during each run, (1) active shoreline segments continually migrated laterally and were never more than momentarily stationary, and (2) ABT showed a tendency to migrate landward with insignificant landward oscillations.
Figure 5. Sequential photo images of Run F1, during which a delta was unable to prograde beyond a submerged overfall. Note that soon after the delta toe reached the overfall, a single large channel stabilized in the axial part of the delta plain. Red lines indicate active shoreline segments.
Figure 6. Trajectory diagrams of active shoreline segments (left column) and ABT (right column) obtained from F series runs. In each run, (1) the behavior of active shoreline segments changed drastically soon after the delta toe had reached the overfall, and (2) ABT simultaneously came to a halt after insignificant basinward migration.
Figure 7. An image of the delta built in Run F4 taken well after the delta toe had reached the overfall. Note the presence of well-developed paired terraces along the axial channel. There is a clear tendency that (1) higher (i.e. older) terraces had higher slopes, and thus (2) the present active river bed has the lowest slopes at any time in the evolution of the alluvial system. These terraces are autogenic since the dynamic forcing \((Q_w, Q_s, \text{base level})\) was kept constant during the run.
Figure 8. Sequential photo images of Run M1, where a delta was built with constant base-level fall. Note that a particular delta lobe extended linearly basinward without any significant lateral shifting.
Figure 9. Sequential photo images of Run M2, where a deltaic lobe extended over 3 m basinward with constant base-level fall as in Run M1, but in a larger tank.
Figure 10. Trajectory diagrams of active shoreline segments (left) and ABT (right) obtained from M series runs. In each run, (1) the lateral extent of migration of active shoreline segments progressively converged to the axial position ($\theta \sim 90^\circ$), and (2) the landward migration of ABT decelerated and came to a stop.
Figure 11. Dimensionless linear-scale trajectory diagrams of active shoreline segments (left) and ABT (right) obtained from M series runs. It appears that the alluvial system observed in Run M1 represented only a significant early stage of its evolution toward the graded state.
Figure 12. River deltas built up in different runs of tank experiments conducted by Muto et al. (2011), which are referred to as the ‘BW series’ in the present paper. The objective of this series was to clarify the control of local basin water depth on the growth of the deltaic body. For this objective, a differential basement was created by placing a platform on the horizontal bottom surface. Basin water depth was 7.2 cm on the platform and 0.7 cm elsewhere. Run BW1 was the reference run of this series, which was conducted with a uniform basin water depth of 7.2 cm. In each of Runs BW1–4, the maximum thickness of the final alluvial deposit was no more than 4 cm; thus, the differential water depth must have exerted a marked influence on delta progradation. Nevertheless, the resulting shoreline configurations look quasi-isotropic, implying that differential bathymetry exerted no critical influence on the overall shoreline configuration. Light concentric lines are projections of horizontal laser lines which were at a vertical interval of 1.0 cm. For detailed experimental conditions, see Muto et al. (2011).
Figure 13. Schematic illustration of a delta assumed in geometrical modeling. See text for a detailed explanation.
Figure 14. Linear relationship between $R_{\text{mig}}$ and $R_{\text{agg}}$ for measurements from Run B_W1. $R_{\text{mig}}$ values were adopted from Muto et al. (2011), and $R_{\text{agg}}$ values were provided by calculation with other parameters including grade index.
Figure 15. Topographic setting of fan deltas/alluvial fans on the Japan Sea coast of central Japan. Note that most of the feeder river mouths have direct connections with submarine canyons of the Toyama Deep Sea Channel. J = Joganjigawa, H = Hayatsukigawa, K = Katakaigawa, Kr = Kurobegawa. Modified from Nakajima et al. (1998).
Figure 16. Downstream profiles of terraces and present river beds. Numerical values for alluvial slopes ($\alpha$) and grade indexes ($G_{index}$) are provided. Note that in each river system higher terraces have higher slopes. Ages of the terraces are based on chronological data by Matsuura et al. (2007) and Fujii et al. (2011).